

Average Bid Method — A Competitive Bidding Strategy

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Abstract: The major drawback of the low bid method, often used for competitive bidding in the U.S. construction industry, is the possibility of awarding a construction contract to a contractor that submits either accidentally or deliberately an unrealistically low bid price. Often, such an occurrence works to the owner's and contractor's detriment by promoting disputes, increased costs, and schedule delays. To address this problem, other countries have adopted the average bid method and award the contract to the contractor whose price is closest to the average of all bids submitted. This paper presents a competitive bidding model for the average bid method and explores its merits relative to the low bid method. The bidding process is analyzed both mathematically and through Monte Carlo simulation. The final results of the average bid model, as well as Friedman's low bid model, are presented in four nomographs that can be used to analyze a competitive situation without the need for any mathematical or numerical manipulation. A comparison of the two methods reveals that the average bid method and its variations have the potential to improve contracting practices both for the owner and the contractor and deserve the industry's increased attention.

Introduction

The competitive bidding process for awarding construction contracts in the U.S. is typically based on the low bid method and is probably as close to pure competition as possible. According to this method, the construction firm submitting the lowest bid receives the right to the construction contract. Its main advantage is that it forces contractors to continuously try to lower costs by adopting cost-saving technological and managerial innovations. These savings are then passed to the owner through the competitive process. When the number of bidders is large, however, as is the case in a slow economy, an owner runs a significant risk of selecting a contractor that has either accidentally or deliberately submitted an unrealistically low price (Grogan 1992). A contractor cannot adhere to such a price and at the same time expect to complete the project according to plans and specifications, and also make a reasonable profit. This often results in excessive claims and disputes during construction that lead to schedule delays, compromises in quality, and increased costs. Several countries, such as Italy and Taiwan, have developed variations of the average bid

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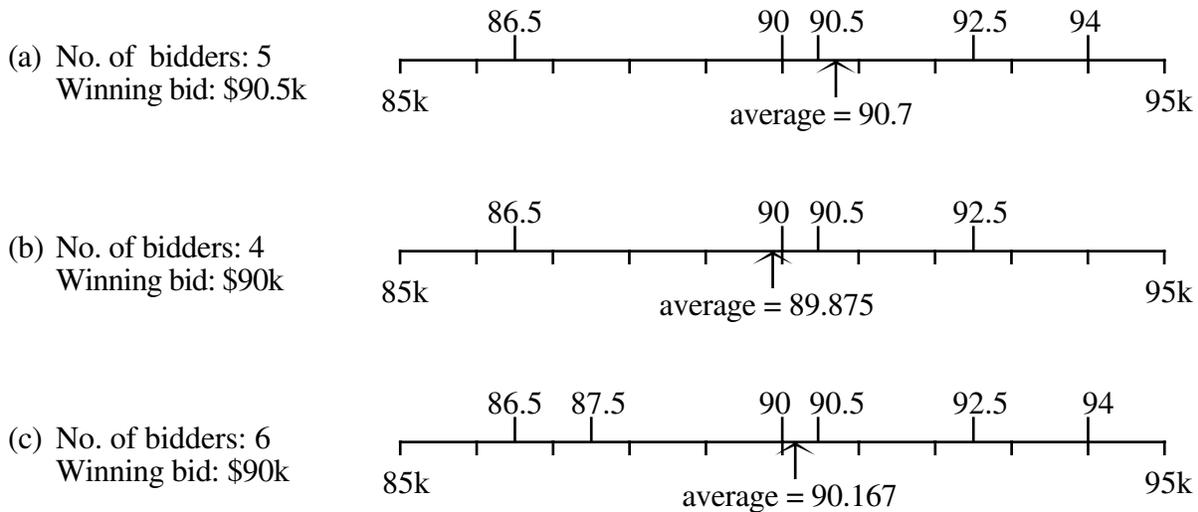


Figure 1: — **Example of the Average Bid Method**

method as an answer to these problems. In Taiwan, for example, the average bid method presented in this paper is rapidly becoming the preferred competitive bidding method.

The objective of this paper is to present a competitive bidding model for the average bid method and to explore its merits relative to the low bid method. The bidding process is analyzed both mathematically and through Monte Carlo simulation. Even though the former is more elegant, the latter is more powerful and can be easily adapted to other variations of the average bid method. The results of the bidding model are presented in four nomographs that can be used to analyze a competitive situation without the need for complicated mathematics. The same graphs present a similar solution for the low bid method based on Friedman's model (Friedman 1956, Ioannou 1988).

The Average Bid Method

In general, the winner based on the average bid method is the contractor whose bid satisfies a certain relationship with the average of all bid prices. Different average bid methods use different procedures for calculating the average, or use different criteria for determining the winning bid. For example, some use an arithmetic average or a weighted average, while others use the average of the remaining bids after all bids that differ more than a certain percentage from the average of all other bids are eliminated. Similarly, the winner might be the contractor whose price is closest to the average, or the contractor whose bid is closest to, but less than the average. The former, for example, is used in Taiwan while the latter is used in Italy. The average bid method presented in this paper is the one used in Taiwan, where the winner is the bidder whose price is closest to arithmetic average of all submitted bids.

To illustrate the mechanism of the average bid method, as well as its differences from the low bid method, consider the example illustrated in Fig. 1. Fig. 1a shows five contractors that have submitted the following bids: 86.5k, 90k, 90.5k, 92.5k, and 94k. The average bid price is 90.7k. The bid of 90.5k is closest to the average and thus wins the project. Fig. 1b shows that if for some reason the bid of 94k had not been submitted, the average would drop to 89.875k, and the winning

bid would have been 90k. Similarly, Fig. 1c illustrates that if a sixth bid of 87.5k were submitted, the average would drop to 90.167k and the winning bid would again be 90k. Thus, even though neither of the two bids at 94k and 87.5k ever win, their presence influences which of the other bids does win. This clearly illustrates the interdependence of all bid prices in determining the winner. In contrast, under the low bid method the winning bid in all three cases would have been the same, 86.5k.

It is clear from this example that the chance of winning is affected by the number of bidders and all submitted bid prices. Consequently, the average bid method is significantly more difficult to model than the low bid method. For the low bid method, the main variable of interest, from a contractor's perspective, is simply the minimum of the bid prices submitted by the other competitors. The contractor's own bid price is not part of the minimum function but rather serves as a value to which the minimum of the opponents' bids is compared. The average bid price, on the other hand, depends on the contractor's own bid price. Furthermore, in order to determine the winner we need to determine the absolute value of the difference of every bid to the average and select the minimum. A change in any of the bid amounts, or the number of bidders, can have a profound effect on any particular bidder's chance of winning.

Basic Definitions

The notation adopted in this paper is to indicate random variables and the identity of contractors using capitalized symbols; decision variables and the values of random variables are shown using lower-case symbols.

We shall examine the bidding process from the perspective of a particular contractor A_0 who is bidding on a new project against n competitors A_1, A_2, \dots, A_n . Let C_i be the project cost estimate and B_i the bid price for competitor $A_i, i = 0, 1, \dots, n$.

Contractor A_0 has already determined a cost estimate c_0 and needs to decide on a bid price b_0 . This is the decision variable of primary interest. Even though there is nothing uncertain about either of these quantities, the actual cost of the project, given that A_0 wins the project, is of course uncertain.

In preparing a cost estimate, contractor A_0 has to make a multitude of decisions: select crew compositions, estimate quantities, estimate production, etc. In fact, the entire estimating process is a series of decisions based on available information. A change in any of these will have a direct effect on the cost estimate c_0 .

After the cost estimate is established, contractor A_0 must choose a markup m_0 to arrive at a bid price $b_0 = c_0 + m_0$. Equivalently, contractor A_0 may select a bid-to-cost ratio $x_0 = 1 + m_0/c_0$ and multiply it by his cost estimate c_0 :

$$b_0 = x_0 c_0 \tag{1}$$

Obviously, contractor A_0 has complete control over the values of c_0, m_0, x_0 , and b_0 . All are decision variables, even if their values have not been determined yet.

A_0 does not know the competitors' cost estimates C_i and bids B_i . Given this state of information, these are considered to be random variables, even though C_i and B_i are decision variables to the corresponding contractor A_i . Furthermore, A_0 's state of information, and hence his perspective, do not change depending on whether contractor A_i has, or has not, already determined and thus fixed the values of C_i and B_i .

The ratio of contractor A_i 's bid B_i to contractor A_0 's cost estimate c_0 will be shown as X_i :

$$X_i = \frac{B_i}{c_0} \quad i = 1, 2, \dots, n \quad (2)$$

It should be noted that for contractor A_0 the ratio $x_0 = b_0/c_0$ is a decision variable, whereas the X_i ($i = 1, \dots, n$) are random variables representing the apparent bid-to-cost ratios for each of the n other competitors.

Friedman's Propositions

To facilitate a direct comparison, the following model for the analysis of the average bid method is based on the same two assumptions as Friedman's model for the low bid method (Friedman 1956, Ioannou 1988):

1. In order to eliminate the effect of the project size, each opponent's bid B_i ($i = 1, 2, \dots, n$) is standardized by taking its ratio to contractor A_0 's cost estimate c_0 . The resulting ratios X_i ($i = 1, 2, \dots, n$) are assumed to be mutually independent.
2. The probability distribution of each apparent bid-to-cost ratio X_i ($i = 1, 2, \dots, n$) does not depend on the values of x_0 and c_0 that A_0 chooses:

$$P[X_i < x_i | x_0, c_0] = P[X_i < x_i] \quad i = 1, 2, \dots, n \quad (3)$$

The theoretical and practical ramifications of these assumptions are examined in detail elsewhere and are not presented here (Ioannou 1988).

Mathematical Formulation

The formal definition for the event "Contractor A_0 wins" when using the average bid method can be stated as follows:

$$\{A_0 \text{ wins} | b_0\} = \left\{ \bigcap_{i=1}^n (|b_0 - \bar{B}| < |B_i - \bar{B}|) \right\} \quad (4)$$

where $\bar{B} = (b_0 + B_1 + \dots + B_n)/(n + 1)$ is the average of the bid prices. Notice that since B_1, B_2, \dots, B_n are random variables, so is \bar{B} . Furthermore, since b_0 is a decision variable, the value of \bar{B} can be influenced by contractor A_0 .

It should be noted here that the average bid method, as defined above, does not apply to the case of only two bidders, A_0 and A_1 . The average $\bar{B} = (b_0 + B_1)/2$ is exactly midway between b_0 and B_1 , and thus there is no way to determine the winner. The following analysis assumes that the competitive bidding process involves more than two competitors.

The general formula for the probability that contractor A_0 wins is:

$$P[A_0 \text{ wins} | b_0] = P[(|b_0 - \bar{B}| < |B_1 - \bar{B}|) \cap \dots \cap (|b_0 - \bar{B}| < |B_n - \bar{B}|)] \quad (5)$$

$$= P\left[\bigcap_{i=1}^n (|b_0 - \bar{B}| < |B_i - \bar{B}|)\right] \quad (6)$$

$$= P\left[\bigcap_{i=1}^n (|x_0 - \bar{X}| < |X_i - \bar{X}|)\right] \quad (7)$$

in which $X_i = B_i/c_0$, and $\bar{X} = (x_0 + X_1 + \dots + X_n)/(n + 1)$.

The simplest application of this formula is the case where contractor A_0 faces only two opponents, A_1 and A_2 . In order to win, A_0 must select a bid-to-cost ratio x_0 that satisfies the following two inequalities:

$$|x_0 - \bar{X}| < |X_1 - \bar{X}| \quad (8)$$

$$|x_0 - \bar{X}| < |X_2 - \bar{X}| \quad (9)$$

Replacing \bar{X} with $(x_0 + X_1 + X_2)/3$ gives:

$$|(2x_0 - X_1 - X_2)| < |(2X_1 - x_0 - X_2)| \quad (10)$$

$$|(2x_0 - X_1 - X_2)| < |(2X_2 - x_0 - X_1)| \quad (11)$$

Squaring both sides of each inequality eliminates absolute values, and collecting terms:

$$3x_0^2 - 3X_1^2 < 6x_0X_2 - 6X_1X_2 \quad (12)$$

$$3x_0^2 - 3X_2^2 < 6x_0X_1 - 6X_1X_2 \quad (13)$$

Finally, extracting common factors produces the inequalities:

$$(x_0 - X_1)(x_0 + X_1) < 2X_2(x_0 - X_1) \quad (14)$$

$$(x_0 - X_2)(x_0 + X_2) < 2X_1(x_0 - X_2) \quad (15)$$

In order for contractor A_0 to win, the random variables X_1 and X_2 must take on values that satisfy both inequalities (14) and (15). Notice that the terms $(x_0 - X_1)$ and $(x_0 - X_2)$ appear on both sides of the above inequalities. They cannot be eliminated, however, because their sign determines the direction of inequality.

The region where the pairs (x_1, x_2) satisfy inequalities (14) and (15) is indicated by the cross-hatched areas in Fig. 2. According to Friedman's assumptions, X_1 and X_2 are iid (independent and identically distributed) with a common probability density function, $f_X(x)$, and a common cumulative distribution function, $F_X(x)$. Integrating $f_X(x)$ over this discontinuous region produces the following probability of winning:

$$P[A_0 \text{ wins} | x_0] = \int_{-\infty}^{x_0} f_X(x_1) dx_1 \int_{x_0}^{+\infty} f_X(x_2) dx_2 + \int_{x_0}^{+\infty} f_X(x_1) dx_1 \int_{-\infty}^{x_0} f_X(x_2) dx_2 \quad (16)$$

$$= 2F_X(x_0)(1 - F_X(x_0)) \quad (17)$$

It can be easily shown that this result is the same as the probability that the value x_0 lies between X_1 and X_2 . In fact, a simple examination shows that the pairs (x_1, x_2) defined by the cross-hatched areas in Fig. 2 are indeed those that bracket x_0 . This is as expected, since this is the necessary and sufficient condition for x_0 to win. If x_0 lies between x_1 and x_2 it will certainly be closest to the average.

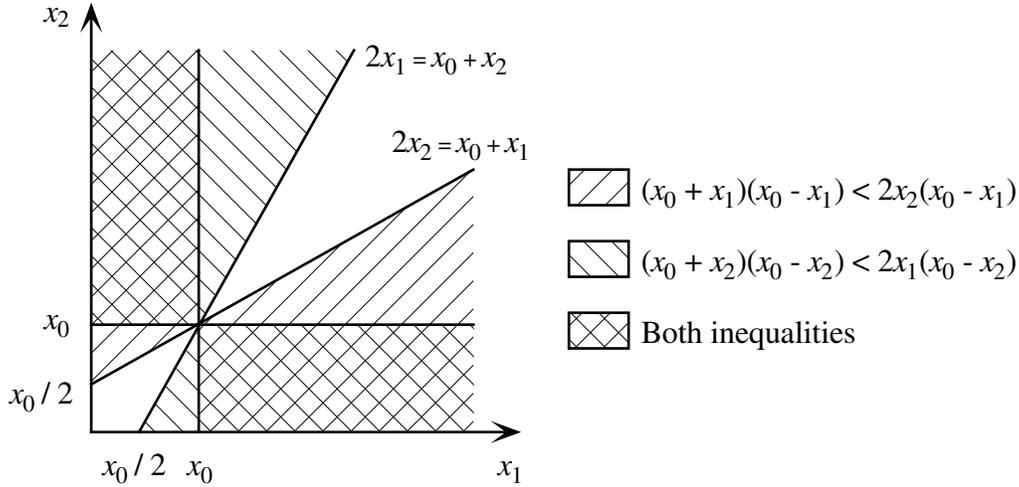


Figure 2: — **Region for Winning Against Two Opponents**

The estimated profit is simply given by $v_0 = b_0 - c_0$. Its expected value is given by:

$$E[V] = P(A_0 \text{ wins})(b_0 - c_0) \tag{18}$$

$$= 2F_X(x_0)(1 - F_X(x_0))(b_0 - c_0) \tag{19}$$

$$= 2F_X(x_0)(1 - F_X(x_0))(x_0 - 1)c_0 \tag{20}$$

The optimum bid-to-cost ratio x_0 can be determined by maximizing Eq. 20.

In general, the number of opponents facing contractor A_0 will be $n > 2$. In order to win the project, A_0 's bid-to-cost ratio x_0 must satisfy the following n inequalities:

$$|x_0 - \bar{X}| < |X_1 - \bar{X}| \tag{21}$$

$$|x_0 - \bar{X}| < |X_2 - \bar{X}| \tag{22}$$

⋮

$$|x_0 - \bar{X}| < |X_n - \bar{X}| \tag{23}$$

Using the same algebra as in the case of two opponents, these can be simplified to:

$$(n - 1)(x_0 - X_1)(x_0 + X_1) < 2(x_0 - X_1)(X_2 + X_3 + \dots + X_n) \tag{24}$$

$$(n - 1)(x_0 - X_2)(x_0 + X_2) < 2(x_0 - X_2)(X_1 + X_3 + \dots + X_n) \tag{25}$$

⋮

$$(n - 1)(x_0 - X_n)(x_0 + X_n) < 2(x_0 - X_n)(X_1 + X_2 + \dots + X_{n-1}) \tag{26}$$

Notice again that even though the terms $(x_0 - X_i)$ appear on both sides of the above inequalities they cannot be eliminated because their sign determines the direction of inequality.

In order for contractor A_0 to win, the random variables X_1, X_2, \dots, X_n must take on values that satisfy inequalities (24)-(26). The region where points (x_1, x_2, \dots, x_n) satisfy inequalities (24)-(26) must be visualized in multidimensional coordinates. Since this is not practical, the probability of winning can be computed by using the Total Probability Theorem. This approach involves

the definition of a set of regions on the one-dimensional axis of real numbers that satisfy all the possibilities implied by the above set of inequalities. This is a very tedious task that leads to integral equations whose limits are linear functions of x_0, x_1, \dots, x_n . These integrals, however, do not provide a closed-form solution for the general probability of winning. This probability can only be computed by assuming a particular probability distribution $f_X(x_i)$ and performing numerical integration. The same solution can be achieved much easier by using simulation as described below.

Simulation Approach

Monte Carlo simulation can be used to determine the probability of winning and to select the optimum bid-to-cost ratio x_0 for a given distribution $f_X(x_i)$. We shall assume that the apparent bid-to-cost ratios, X_1, X_2, \dots, X_n , are iid, following a Normal distribution with mean m_X , and variance σ_X^2 .

In order to arrive at a numerical solution that is independent of m_X and σ_X^2 , the following standardized variables are defined:

$$m'_X = \frac{m_X - 1}{\sigma_X} \quad (27)$$

$$x'_0 = \frac{x_0 - m_X}{\sigma_X} \quad (28)$$

The main input to the simulation process consists of the number of competitors n , the number of simulation sets s , and the number of projects per simulation set m . The results presented here are based on $s = 100$, and $m = 1000$.

For each project $j = 1, \dots, m$, in each simulation set $k = 1, \dots, s$, we sample n standardized apparent bid-to-cost ratios x'_{ijk} ($i = 1, \dots, n$) from the standard Normal distribution $N(0,1)$. Notice that the index i represents a competitor and the index j represents a project. Index k represents the simulation set and may be ignored for the time being.

For a given project (j, k) we compute the average \bar{x}'_{jk} of the n competitors' values x'_{ijk} and a range around the average defined by a lower bound l_{jk} and an upper bound h_{jk} . The range (l_{jk}, h_{jk}) defines the region within which x'_{0jk} wins project (j, k). Its bounds are determined by

$$h_{jk} = \min(\bar{x}'_{jk} + d^+, \bar{x}'_{jk} + \frac{n+1}{n-1}d^-) \quad (29)$$

$$l_{jk} = \max(\bar{x}'_{jk} - d^-, \bar{x}'_{jk} - \frac{n+1}{n-1}d^+) \quad (30)$$

where d^+ is the absolute value of the distance from the average \bar{x}'_{jk} to the next higher x'_{ijk} , and d^- is the absolute value of the distance from the average \bar{x}'_{jk} to the next lower x'_{ijk} . The derivation of these bounds is shown in Appendix II.

Any value x'_{0jk} in the region (l_{jk}, h_{jk}) wins project (j, k). The best selection, however, is $x'_{0jk} = h_{jk}$ since it not only wins the project but also maximizes profit.

Repeating this process for a given simulation set k , results in m ranges (l_{jk}, h_{jk}) , one for each project. We now determine how many of the m projects in the current simulation set would have been won by each of the m candidate values $x'_{0jk} = h_{jk}$. This is determined by comparing each candidate value h_{jk} to all ranges (l_{qk}, h_{qk}) , where $q = 1, \dots, m$. Project q is considered won

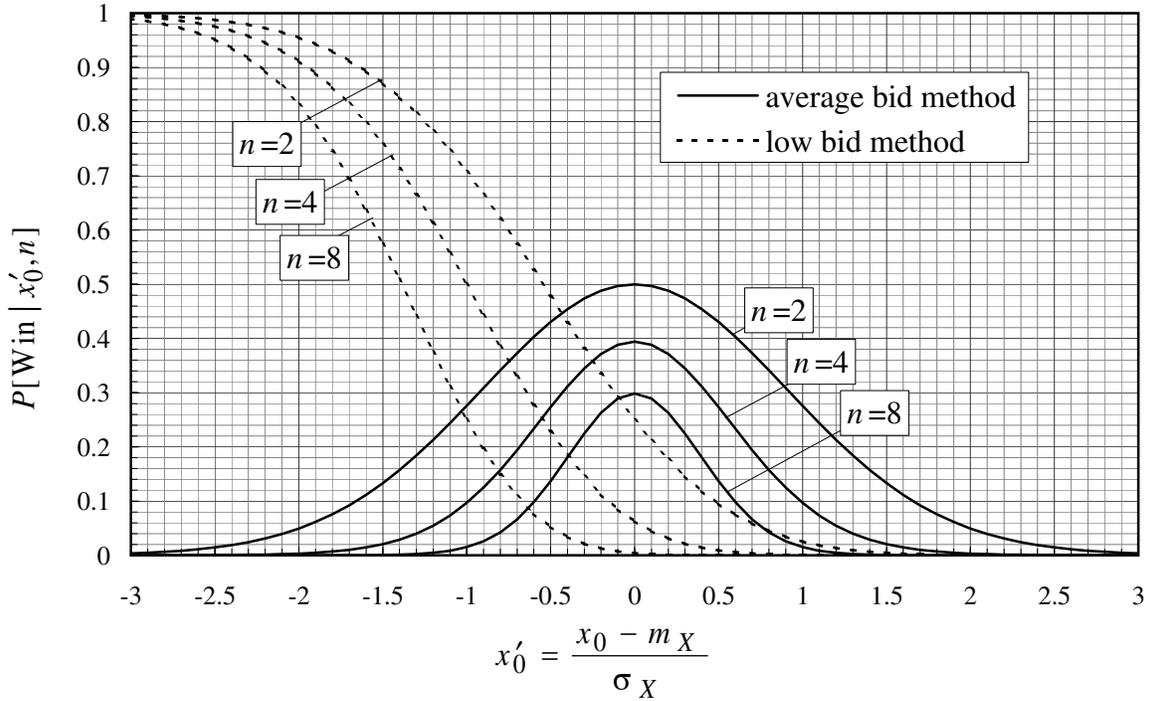


Figure 3: — **Probability of Winning as a function of x'_0, n**

if the selected h_{jk} is within the project's corresponding range (l_{qk}, h_{qk}) . Given that we use the standardized bid-to cost ratio $x'_{0jk} = h_{jk}$, the probability of winning any given project, p_{jk} , equals the number of projects won divided by m (Fig. 3).

To determine the optimum bid-to-cost ratio we must first select a particular value for the standardized mean m'_X and then compute the m corresponding expected profits:

$$\frac{v_{jk}}{c_0\sigma_X} = p_{jk} \frac{x_{0jk} - 1}{\sigma_X} \quad (31)$$

$$= p_{jk}(x'_{0jk} + m'_X) \quad (32)$$

$$= p_{jk}(h_{jk} + m'_X) \quad (33)$$

where $j = 1, \dots, m$. Of the resulting m profits the maximum defines the optimum quantity

$$\frac{x_{0k}^* - 1}{\sigma_X} = (h_{jk}^* + m'_X) \quad (34)$$

that corresponds to the chosen value for the standardized mean m'_X . To create the graphs shown in Fig. 4 we select different values for m'_X and compute the corresponding $(x_{0k}^* - 1)/\sigma_X$.

To improve accuracy, the actual $(x_{0k}^* - 1)/\sigma_X$ in Fig. 4 represent the average of the s values $(x_{0k}^* - 1)/\sigma_X$ derived from all simulation sets for the same standardized mean m'_X . The same is true for $P[\text{Win}|x'_0, n]$ in Fig. 3, $E[V^*]/(c_0\sigma_X)$ in Fig. 5, and $P[\text{Win}|x'_{0k}, m'_X, n]$ in Fig. 6.

As a check, the same results have also been obtained by using regression analysis to approximate the probability of winning $P[\text{Win}|x'_0, n]$ in Fig. 3 as a function of x'_0 , and n . Once the probability of

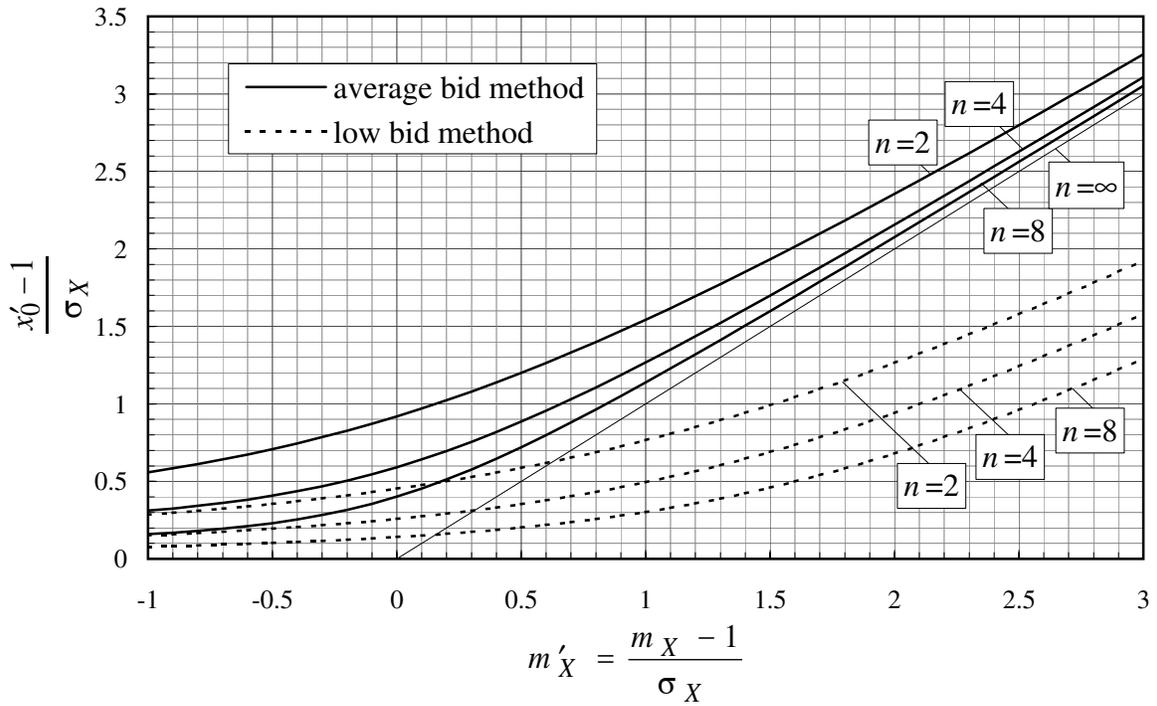


Figure 4: — Optimum Markup as a function of m'_X, n

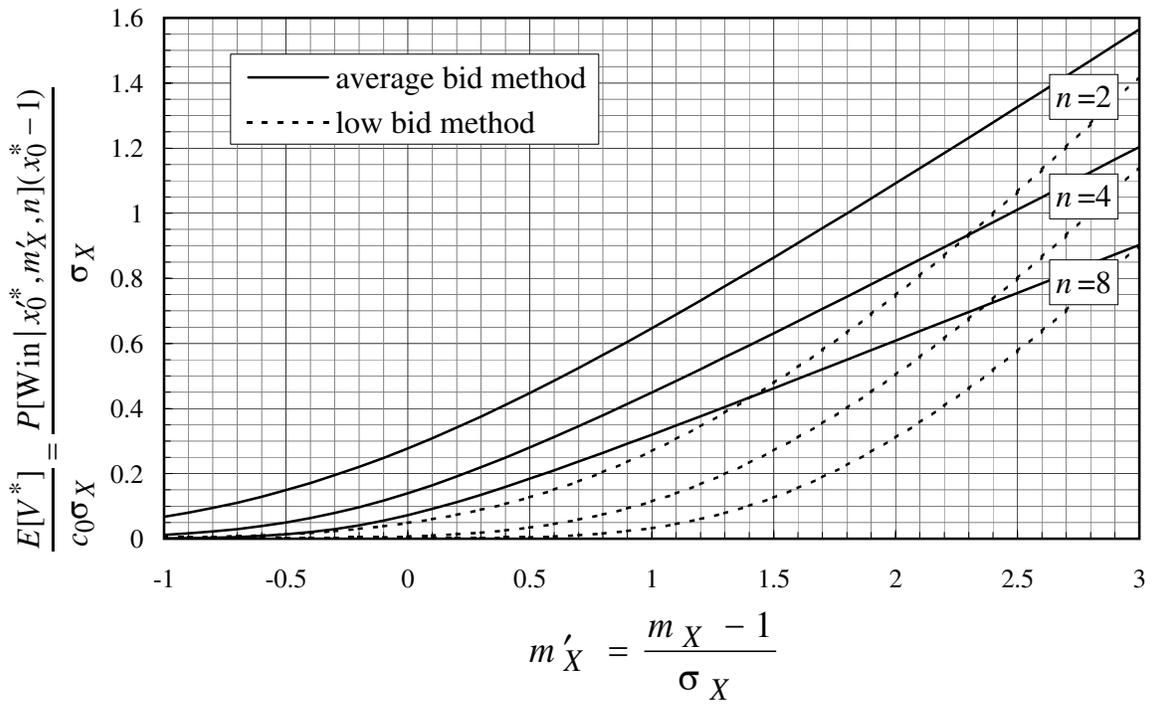


Figure 5: — Optimum Expected Profit as a function of m'_X, n

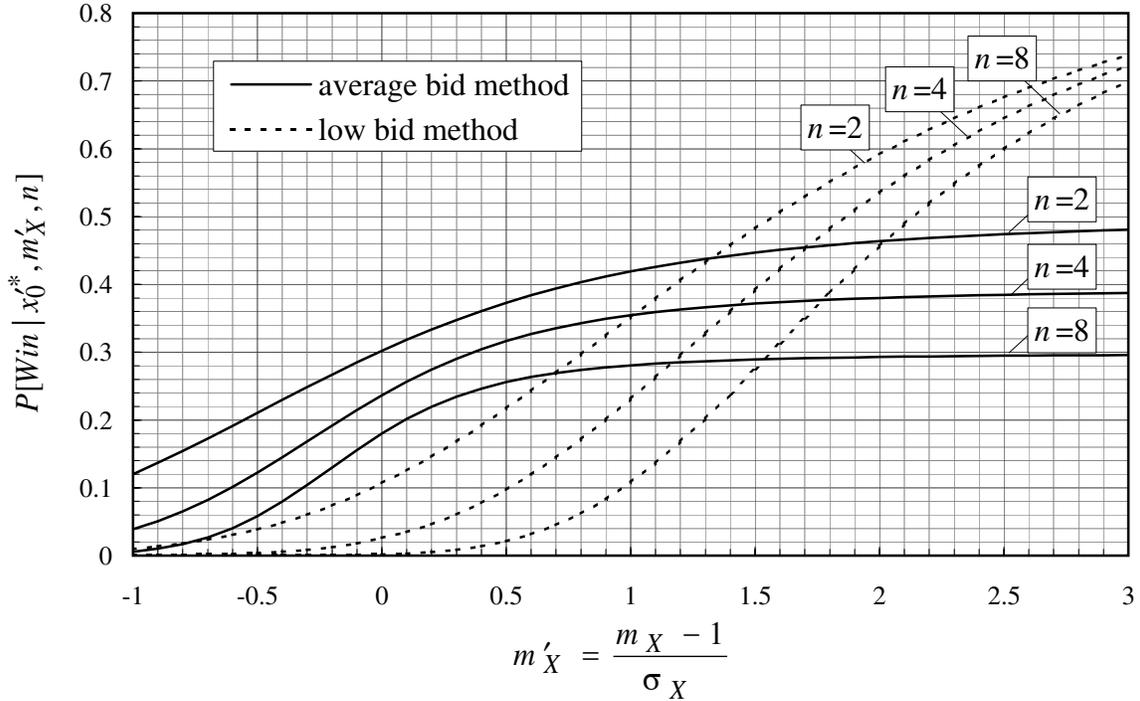


Figure 6: — **Probability of Winning as a function of m'_X, n**

winning is known analytically, the optimum bid-to-cost ratio can be determined by using calculus in a manner similar to that outlined in Appendix III. Appendix III presents the methodology for determining the optimum bid-to-cost ratio using Friedman's model for the low bid method. Both the simulation and analytical approaches produce the same results as shown in Fig. 3-6.

Example Application

The graphs for either the average bid method or the low bid method, require that a contractor use past data to produce estimates \hat{m}_X and $\hat{\sigma}_X^2$ for the Normal distribution's mean and variance:

$$\hat{m}_X = \frac{\sum_{j=1}^N \sum_{i=1}^{n_j} X_{ij}}{\sum_{j=1}^N n_j} \quad (35)$$

$$\hat{\sigma}_X^2 = \frac{\sum_{j=1}^N \sum_{i=1}^{n_j} (X_{ij} - \hat{m}_X)^2}{\sum_{j=1}^N n_j} \quad (36)$$

The historical bid-to-cost ratios are given by $X_{ij} = B_{ij}/c_{0j}$, where, for each project j , B_{ij} is the bid submitted by the i th contractor, c_{0j} is the estimated cost for contractor A_0 , and n_j is the number of competitors facing A_0 . N is the total number of projects for which the above data are available.

To illustrate the use of these figures for the average bid method, let us assume that contractor A_0 is bidding against $n = 4$ opponents, and that the mean and variance have been estimated to be $m_X = 1.1$ and $\sigma_X = 0.1$. The standardized mean is $m'_X = (1.1 - 1)/0.1 = 1$. From Fig. 4 we find

$(x_0^* - 1)/\sigma_X = 1.27$, which leads to the optimum bid-to-cost ratio $x_0^* = 1.127$, or a markup of 12.7%. Fig. 5 shows that the expected profit for this markup is $E[V^*] = 0.45\sigma_X c_0 = 0.045c_0$, or 4.5% of the estimated cost. Finally, Fig. 6 provides us with the probability of winning at the optimum markup of 12.7%, $P[\text{Win}|x_0^*, m'_X, n] = 0.353 = 35.3\%$. Notice that the same probability can also be obtained from Fig. 3 for $x'_0 = (1.127 - 1.1)/0.1 = 0.27$. The use of Fig. 3-6 to determine the same quantities for the low bid method is completely analogous.

The Average vs the Low Bid Method

It is apparent that the results obtained from the model for the average bid method are significantly different from those for the low bid method. These differences essentially fall into four groups.

The first concerns the probability of winning given a standardized bid-to-cost ratio x'_0 , as shown in Fig. 3. It should be noted that the curves in this figure should not be confused with probability density functions, even though they appear to be very similar to the Normal distribution. The curves for the average bid method are symmetrical around $x'_0 = 0$. The maximum probability of winning always occurs at $x'_0 = 0$, irrespective of the number of opponents n . The maximum value for the probability of winning does not exceed 0.5 and this value occurs only when $n = 2$. In contrast, the curves for the low bid method are asymmetrical. The maximum value for the probability of winning approaches 1.0 at very low values of x'_0 , no matter how many opponents a contractor expects. This implies that a contractor can guarantee winning the project by cutting down his bid, something that simply cannot be done when using the average bid method. This is probably the major reason why certain owners may prefer the average bid method. It provides a safeguard against very low bid prices, be they accidental or deliberate.

The second difference concerns the optimum standardized bid-to-cost ratio $(x_0^* - 1)/\sigma_X$ for a given standardized mean m'_X , as shown in Fig. 4. Given the same m'_X and n , this ratio is always larger when using the average bid method. As the number of opponents n becomes very large the curves for the average bid method approach a 45° straight line through the origin. This indicates that the optimum bid-to-cost ratio x_0^* approaches the mean m_X . The same trend is observed when m'_X becomes large, irrespective of the number of bidders. This should be expected because a large value for m'_X implies a very small variance σ_X . This means that the opponents' bids are closely clustered around the mean. The only way for a contractor to win is to select the mean bid-to-cost ratio no matter how many opponents he faces. In contrast, the curves for the low bid method do not merge but become parallel straight lines for large values of m'_X . As the number of opponents n increases, these curves approach the X axis, which implies that the contractor should bid the project at cost. Thus, for a large number of competitors, an owner using the average bid method should expect to sign a contract for a fair and reasonable price as established by the consensus of the industry, whereas an owner using the low bid method should expect an unrealistically low price that is bound to lead to problems later.

The third difference concerns the maximum expected profit given a particular m'_X , as shown in Fig. 5. For the same number of opponents n , this profit is always higher when using the average bid method. This explains why this method might be favored by contractors. As expected, when n becomes very large, the optimum expected profit for either method is zero. Notice, however, that this does not mean that for both methods the owner should expect to sign a contract at cost. The reason the expected bid approaches zero for the average bid method is because the probability of

winning does so. The bid-to-cost ratio, however, approaches m'_X . For the low bid method, both the probability of winning and the optimum markup approach zero.

The final difference concerns the probability of winning at the optimum bid-to-cost ratio $(x_0^* - 1)/\sigma_X$ for a given m'_X , as shown in Fig. 6. For small values m'_X this probability is larger for the average bid method, whereas for high values it is larger for the low bid method. This is again as expected, since a large variance favors winning under the average bid method, while a small variance favors winning when using the low bid method.

Conclusion

The basic advantage of the average bid method, from an owner's perspective, is that it safeguards against signing a construction contract for an unrealistically low bid price that almost certainly will lead to adversarial relationships during construction (Grogan 1992). Similarly, contractors are protected from having to honor a bid containing a gross mistake or oversight. This is certainly the main drawback of the low bid method. Very low bid prices often lead to excessive claims and disputes, and result in cost increases and schedule delays.

The basic drawback of the average bid method as defined here is that it does not necessarily promote price competition that leads to lower costs for the owner. It is easy to see that a technological or managerial breakthrough that results in major cost savings will not necessarily be passed on to the owner in the form of lower prices, unless this breakthrough is known to be available to all bidders.

Contrary to first impressions, the average bid method does not discourage innovation, technology development and adaptation, and cost reduction. Any such savings, when not available to all bidders, may not give the contractor a competitive advantage, but do result in significantly higher profits in projects won. When such savings are available throughout the industry, however, bid prices should be expected to gradually fall and the savings will eventually be passed to the owner.

It is this mechanism that suggests more effective variations of the average bid method. For example, it would probably be more effective to award the project to the bid closest to, but less than the average, an approach used in Italy as well as in Taiwan. This approach would lead to optimum bid-to-cost ratios that are slightly less than the mean. The curves for the average bid method presented in Fig. 4 lie above the straight line for $n = \infty$ and thus imply that the optimum bid-to-cost ratio should be slightly higher than the mean. Under ideal conditions, where all bidders use the results presented here, this would slowly result in a price increase to the owner. The elimination of bids that exceed the mean, however, should negate this effect and produce curves that are on the lower side of $n = \infty$. Notice that for very small values of the mean m'_X , the curves for both methods in Fig. 4 call for bid-to-cost ratios that are greater than m'_X . This illustrates the counter effect of profitability. Contractors should not lower their prices to the point of losing money, even if doing so lowers the probability of winning.

Although attractive, the average bid method is not without its pitfalls. As is the case for the low bid method, collusion among the bidders and the absence of prequalification can negate its intent and produce undesirable results. For example, it has come to the authors' attention, that in a certain country contractors try to obtain a competitive advantage by forming several dummy construction companies that bid the same projects as their affiliated real contractor. These dummy companies and the contractor submit bid prices that are very close to each other and thus pull the overall average towards their own price. If a dummy company wins the project it simply passes the

entire project to the affiliated contractor.

In conclusion, it is evident from this discussion that the development of more efficient competitive bidding methods for the U.S. construction industry is a subject that deserves significant attention. A departure from the low bid method as practiced today has the potential to improve the longevity of construction firms, the elimination of accidentally or deliberately low bid prices, and the improvement of relationships between owner and contractor during construction. Furthermore, it should result in increased and fair profitability for contractors. Indeed, the low profitability of construction firms due to intense competition was recently cited by the president of a large U.S. construction company in his speech at Construction Congress '91 as the major reason why U.S. contractors cannot afford to invest in research and development at the levels enjoyed by Japanese firms.

Appendix I. — References

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Appendix II. — Upper and Lower Bounds Used in Simulation

Let contractor A_0 compete against n opponents whose standardized bid-to-cost ratios are known to be $x'_i, i = 1, 2, \dots, n$. Our objective is to define a region (l, h) with upper and lower bounds h and l within which A_0 's standardized bid-to-cost ratio x'_0 wins the project.

Let \bar{x}' be the average of the standardized bid-to-cost ratios submitted by the n opponents:

$$\bar{x}' = \frac{1}{n} \sum_{i=1}^n x'_i \quad (37)$$

Notice that the average \bar{x}' does not include x'_0 . Let us define \bar{x}'_0 to be the average including x'_0 :

$$\bar{x}'_0 = \frac{1}{n+1} \sum_{i=0}^n x'_i \quad (38)$$

$$= \frac{n\bar{x}' + x'_0}{n+1} \quad (39)$$

Of the n values x'_i , let x'_h be the one that is higher than but closest to the average \bar{x}' . Similarly, let x'_l be the one that is lower than but closest to the average \bar{x}' . Let d^+ be the absolute value of the distance from the average \bar{x}' to x'_h , and d^- be the absolute value of the distance from the average \bar{x}' to x'_l .

First, we shall show that x'_0 cannot win the project if it is so high that \bar{x}'_0 is greater than x'_h . Let us determine the requirement for x'_0 so that \bar{x}'_0 is indeed larger than x'_h :

$$\bar{x}'_0 > x'_h \quad (40)$$

$$\frac{n\bar{x}'_0 + x'_0}{n+1} > x'_h \quad (41)$$

$$x'_0 > (n+1)x'_h - n\bar{x}'_0 \quad (42)$$

It is easy to show that because of inequality (42) x'_0 must also be greater than \bar{x}'_0 . Thus, to win the project, x'_0 must satisfy:

$$x'_0 - \bar{x}'_0 < \bar{x}'_0 - x'_h \quad (43)$$

$$x'_0 - \frac{n\bar{x}'_0 + x'_0}{n+1} < \frac{n\bar{x}'_0 + x'_0}{n+1} - x'_h \quad (44)$$

$$nx'_0 - n\bar{x}'_0 < n\bar{x}'_0 + x'_0 - (n+1)x'_h \quad (45)$$

$$(n-1)x'_0 < 2n\bar{x}'_0 - (n+1)x'_h \quad (46)$$

$$x'_0 < \frac{2n}{n-1}\bar{x}'_0 - \frac{n+1}{n-1}x'_h \quad (47)$$

Combining inequalities (42) and (47) produces:

$$(n+1)x'_h - n\bar{x}'_0 < \frac{2n}{n-1}\bar{x}'_0 - \frac{n+1}{n-1}x'_h \quad (48)$$

$$(n^2-1)x'_h - (n^2-n)\bar{x}'_0 < 2n\bar{x}'_0 - (n+1)x'_h \quad (49)$$

$$(n^2-1+n+1)x'_h < (n^2-n+2n)\bar{x}'_0 \quad (50)$$

$$x'_h < \bar{x}'_0 \quad (51)$$

which is impossible by the definition of x'_h . As a result, our original assumption is false, and in order for x'_0 to win the project the following must be true,

$$\bar{x}'_0 < x'_h \quad (52)$$

which requires that:

$$x'_0 < (n+1)x'_h - n\bar{x}'_0 \quad (53)$$

$$x'_0 < x'_h + nd^+ \quad (54)$$

Furthermore, in order for x'_0 to win:

$$x'_0 < x'_h < x'_h + nd^+ \quad (55)$$

Otherwise, x'_h would be closer to the average \bar{x}'_0 and would win the project.

Similarly, we can show that the following two inequalities must also be true:

$$\bar{x}'_0 > x'_i \quad (56)$$

$$x'_0 > x'_i \quad (57)$$

Thus, the following conditions are necessary for x'_0 to win:

$$x'_l < \bar{x}'_0 < x'_h \quad (58)$$

$$x'_l < x'_0 < x'_h \quad (59)$$

However, they are not sufficient. In order for x'_0 to win, it must also beat both x'_h and x'_l , and thus be the closest to the average \bar{x}'_0 .

For x'_0 to beat x'_l :

$$x'_0 - \bar{x}'_0 < \bar{x}'_0 - x'_l \quad (60)$$

$$x'_0 - \frac{n\bar{x}' + x'_0}{n+1} < \frac{n\bar{x}' + x'_0}{n+1} - x'_l \quad (61)$$

$$nx'_0 - n\bar{x}' < n\bar{x}' + x'_0 - (n+1)x'_l \quad (62)$$

$$(n-1)x'_0 < 2n\bar{x}' - (n+1)(\bar{x}' - d^-) \quad (63)$$

$$x'_0 < \bar{x}' + \frac{n+1}{n-1}d^- \quad (64)$$

$$\bar{x}' - d^- < x'_0 < \bar{x}' + \frac{n+1}{n-1}d^- \quad (65)$$

Similarly, for x'_0 to beat x'_h :

$$\bar{x}'_0 - x'_0 < x'_h - \bar{x}'_0 \quad (66)$$

$$\frac{n\bar{x}' + x'_0}{n+1} - x'_0 < x'_h - \frac{n\bar{x}' + x'_0}{n+1} \quad (67)$$

$$n\bar{x}' - nx'_0 < (n+1)x'_h - n\bar{x}' - x'_0 \quad (68)$$

$$2n\bar{x}' - (n+1)(\bar{x}' + d^+) < (n-1)x'_0 \quad (69)$$

$$\bar{x}' - \frac{n+1}{n-1}d^+ < x'_0 \quad (70)$$

$$\bar{x}' - \frac{n+1}{n-1}d^+ < x'_0 < \bar{x}' + d^+ \quad (71)$$

The two lower bounds and two upper bounds given by inequalities (65) and (71) define a region as illustrated by the shaded area in Fig. 7. Given a value \bar{x}' , Fig. 7 shows the interval within which x'_0 must lie in order to win against both x'_l and x'_h . Thus, the necessary and sufficient conditions for A_0 's standardized bid-to-cost ratio x'_0 to win are:

$$l < x'_0 < h \quad (72)$$

where,

$$h = \min(\bar{x}' + d^+, \bar{x}' + \frac{n+1}{n-1}d^-) \quad (73)$$

$$l = \max(\bar{x}' - d^-, \bar{x}' - \frac{n+1}{n-1}d^+) \quad (74)$$

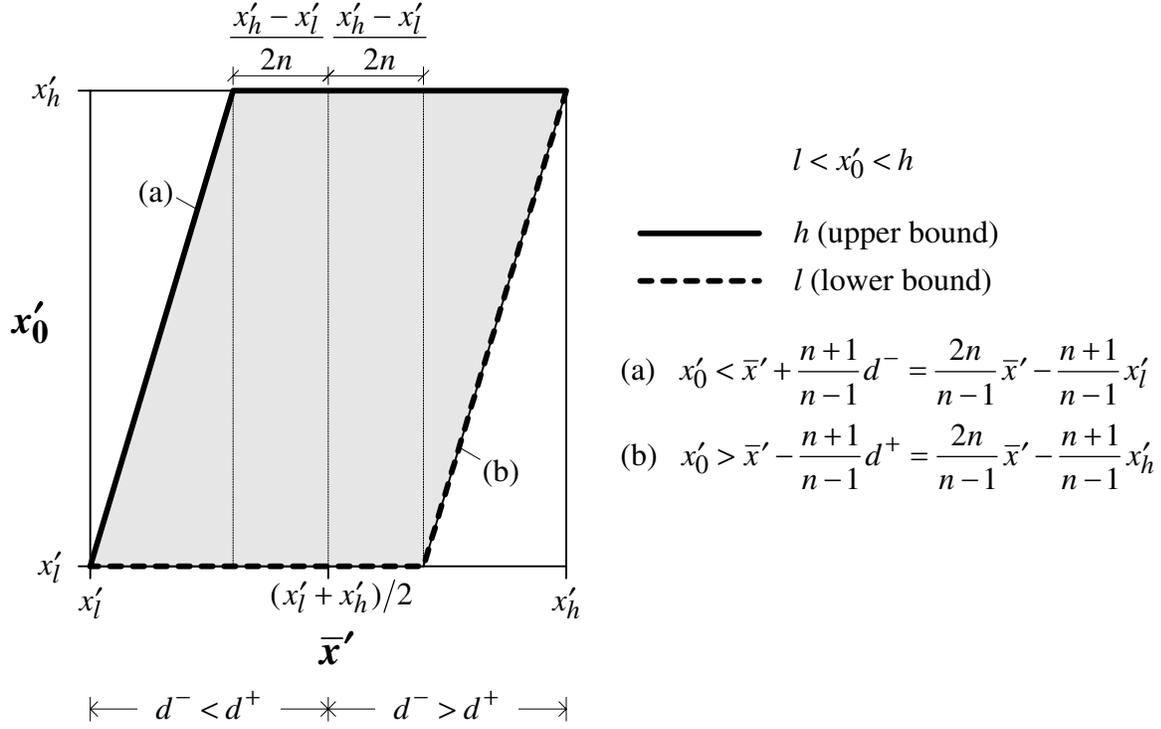


Figure 7: — Winning region for x'_0 as a function of \bar{x}'

Appendix III — Optimum Markup by Direct Solution of Friedman's Model

The following illustrates how to directly solve Friedman's model for the optimum bid-to-cost ratio x_0^* :

$$E[V] = P[\text{Win}|x_0, n](x_0 - 1)c_0 \quad (75)$$

$$\frac{dE[V]}{dx_0} = c_0 P[\text{Win}|x_0, n] + c_0 \frac{dP[\text{Win}|x_0, n]}{dx_0} (x_0 - 1) \quad (76)$$

Setting $dE[V]/dx_0 = 0$, we get:

$$x_0^* - 1 = - \frac{P[\text{Win}|x_0^*, n]}{\frac{dP[\text{Win}|x_0, n]}{dx_0} \Big|_{x_0^*}} \quad (77)$$

This is the basic equation for solving directly for x_0^* . To apply this equation let's use the simple version of Friedman's model:

$$P[\text{Win}|x_0, n] = [1 - F_U(x'_0)]^n \quad (78)$$

where $F_U(u)$ is the standard Normal distribution $N(0, 1)$; $x'_0 = (x_0 - m_X)/\sigma_X$; $m_X = E[X_i]$; $\sigma_X^2 = \text{Var}[X_i]$; and n is the number of opponents. Differentiating Friedman's model yields:

$$\frac{dP[\text{Win}|x_0, n]}{dx_0} = -\frac{n}{\sigma_X} [1 - F_U(x'_0)]^{n-1} f_U(x'_0) \quad (79)$$

Substituting this into the basic equation gives:

$$x_0^* - 1 = \frac{[1 - F_U(x_0'^*)]^n}{\frac{n}{\sigma_X} [1 - F_U(x_0'^*)]^{n-1} f_U(x_0'^*)} \quad (80)$$

$$\frac{x_0^* - 1}{\sigma_X} = \frac{1 - F_U(x_0'^*)}{n f_U(x_0'^*)} \quad (81)$$

This can be simplified to:

$$x_0'^* + m'_X = \frac{1 - F_U(x_0'^*)}{n f_U(x_0'^*)} \quad (82)$$

$$x_0'^* = \frac{Q_U(x_0'^*)}{n f_U(x_0'^*)} - m'_X \quad (83)$$

where $m'_X = (m_X - 1)/\sigma_X$ and $Q(u) = 1 - F_U(u)$ is the complement of the standard Normal CDF. Given m'_X and n , the above nonlinear equation can be solved by selecting an initial value u_0 and performing successive iterations:

$$u_{k+1} = \frac{Q_U(u_k)}{n f_U(u_k)} - m'_X \quad (84)$$

until we achieve the desired level of accuracy ϵ :

$$|u_{k+1} - u_k| < \epsilon \quad (85)$$

At this point we simply set $x_0'^* = u_{k+1}$, and $x_0^* = m_X + \sigma_X u_{k+1}$.

Appendix IV. — Notation

The following symbols are used in this paper:

A_i = The i th competing contractor.

b_0 = The bid price of contractor A_0 .

B_i = The bid price of contractor A_i .

$\bar{B} = (b_0 + B_1 + \dots + B_n)/(n + 1)$.

c_0 = The project cost estimate of contractor A_0 .

C_i = The project cost estimate of contractor A_i .

$F_X(x) = P[X \leq x]$. The cumulative distribution function of the random variable X evaluated at x .

$f_X(x) = P[x \leq X \leq x + dx]/dx$. The probability density function of the random variable X evaluated at x .

m_0 = The markup of contractor A_0 .

m_X = The expected value of X_i .

$$m'_X = (m_X - 1)/\sigma_X.$$

n = The number of competitors bidding against contractor A_0 .

σ_X^2 = The variance of X_i .

$x_0 = b_0/c_0$. The bid-to-cost ratio of contractor A_0 .

$$x'_0 = (x_0 - m_X)/\sigma_X.$$

$X_i = B_i/c_0$. Contractor A_i 's apparent bid-to-cost ratio (as viewed by A_0).

$$\bar{X} = (x_0 + X_1 + \dots + X_n)/(n + 1).$$