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A Geologic Prediction Model For Tunneling

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Abstract: Geologic uncertainty in underground construction often leads to design and construction conservatism and hence to inflated costs. This paper presents a general model for the probabilistic prediction of tunnel geology, as a basis for developing more effective tunnel design and construction decision support systems. The geologic conditions along the tunnel alignment are modeled by a set of geologic parameters (such as rock type, joint density, degree of weathering, etc.), each following a continuous-space, discrete-state Markov process. The state probabilities for each geologic parameter are initially based on general geologic information and are later updated to reflect the outcomes and reliability of the location-specific, non-deterministic observations provided by exploration programs. The resulting posterior geologic parameter profiles are aggregated into a single probabilistic ground class profile that can be used for determining optimal tunnel design and construction strategies. The model is illustrated by an example application.

Introduction

Geologic uncertainty is the primary source of risk in underground construction, often leading to the assumption of the worst possible ground conditions and thus to inflated costs. Significant savings are possible by reducing design and construction conservatism and selecting tunnel excavation and support based on the expected geologic conditions to be encountered during construction.

For a particular geologic profile the optimal sequence of design and construction methods is a

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function of the persistence of the geologic conditions in which each method is technically feasible, the associated production costs, and the cost of changing to different excavation and support methods. As a result, the design, construction planning and cost estimating of tunnels require the explicit consideration of the variability of geologic conditions as they naturally exist at the site.

These conditions are not known prior to construction and must be inferred on the basis of general information describing the geologic formations in the vicinity of the project and location-specific observations provided by subsurface exploration programs. Tunnel designers and contractors must use this information to develop a geologic description of the project that quantifies their inherent state of uncertainty (due to the extent and reliability of available information) and which can also be used for practical decision making.

The above objectives can be achieved by developing a probabilistic description of geology and by then selecting the excavation-support sequence using the tools of operations research and decision theory. This paper describes a geologic prediction model for tunneling projects based on the continuous-space, discrete-state Markov process. The proposed model expands the capabilities of its predecessors by readily incorporating all available geologic information and by automating the procedures for updating geologic predictions based on new observations (Ashley 1981; Chan 1981; Moavenzadeh 1974; Vick 1974). It can also be extended to model the results of future exploration programs using simulation, and has formed the basis for the development of other, more specialized, tunnel design and construction decision support systems (Ioannou 1984, 1985; Kim 1984). Its use is demonstrated by an example application to the Seabrook Power Station discharge tunnel.

Modeling Objectives and Requirements

The general objectives, considerations and requirements for the development of the proposed geologic prediction model can be summarized as follows:

- Geologic prediction is based on information that may be characterized as *location-specific* or *general*. This is to stress that in addition to the direct observations provided by site exploration, knowledge of the regional geology and the geologic history of the area and thorough appreciation and understanding of the way in which rocks respond to changing

geological environments are equally important (Robinson 1972). General information must thus be explicitly incorporated in the model in order to interpret the results of location-specific exploration, and to predict the geology between, or away from, direct observations.

- The general information about a tunnel's geology essentially remains unchanged as exploration proceeds, while the particular information from location-specific observations increases. Geologic predictions must therefore be easily updated to reflect all available observations irrespective of whether the new information is in the form of imperfect observations or in the form of direct geologic predictions based on expert judgment.
- A complete model should be able to include all the geologic parameters that have a strong influence on design and construction decisions. The model must not place constraints on either the number of parameters or the number of parameter states that can be considered. Most importantly, the model should be capable of realistically simulating possible geologic profiles to facilitate overall design and construction planning. The generated profiles should not contradict the general expectations about the project geology or observations on parameter states known before construction and must not deviate considerably from the general geologic conditions in the vicinity of the project.

Tunnel Geology as a Random Process

Design and construction decisions in tunneling depend on such geologic parameters as rock type, joint density, faulting, joint appearance, degree of weathering and groundwater characteristics. In defining a set of design-construction options prior to construction, it is necessary to account for the spatial variability of these parameters along the tunnel alignment. Even though the geologic conditions at every point are predetermined by nature, these conditions are not perfectly known before the construction of the project. As a result, the Bayesian *degree of belief* interpretation of probability can be adopted to model the project's geology as a random process in space (Chan 1981; Krumbein 1969, 1965).

The spatial variability of a geologic parameter (such as *rock type*) can be described by its transitions in and out of parameter states (for example, *granite, schist, limestone*, etc.) each of

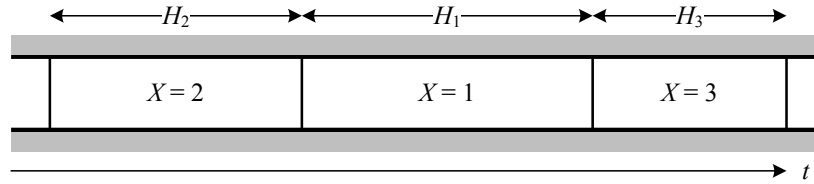


Figure 1: —**Geologic Parameter $X(t)$ as a Random Process Along the Tunnel Axis**

which has a certain length of persistence. From a modeling point of view, any particular geologic parameter can thus be considered as a scalar random process $X_i(t)$ whose state probabilities (PMF) are a function of the distance t from an arbitrary origin, such as the portal of the tunnel (Fig. 1).

A complete description of tunnel geology, however, requires the simultaneous characterization of all the geologic parameters of interest. This can be accomplished by aggregating the individual geologic parameter random processes $X_i(t)$ into the joint vector process:

$$\underline{\mathbf{X}}(t) = [X_1(t), X_2(t), \dots, X_N(t)] \quad (1)$$

where N is the total number of geologic parameters considered. The fundamental objective of a geologic prediction model is the estimation of the vector process $\underline{\mathbf{X}}(t)$ in order to describe the overall variability of tunnel geology.

The distribution of $\underline{\mathbf{X}}(t)$ is the joint probability distribution of all the defined geologic parameters $X_i(t)$ and can be easily evaluated using the conditional expansion:

$$\begin{aligned} P[\underline{\mathbf{X}}(t) = \underline{\mathbf{x}}] &= P[X_1(t) = x_1] \cdot P[X_2(t) = x_2 | X_1(t) = x_1] \cdot P[X_3(t) = x_3 | X_1(t) = x_1, X_2(t) = x_2] \cdots \\ &\cdots P[X_N(t) = x_N | X_1(t) = x_1, X_2(t) = x_2, \dots, X_{N-1}(t) = x_{N-1}] \end{aligned} \quad (2)$$

This decomposition is particularly attractive in geologic modeling, since a physical hierarchy often exists between the components of $\underline{\mathbf{X}}(t)$ —i.e. between the geologic parameters rock type, joint density, permeability, etc. For example, one may argue on physical grounds that joint density $X_2(t)$ depends on rock type $X_1(t)$ and not vice versa, that permeability $X_3(t)$ depends on rock type and joint density and not vice versa, etc., and thus define conditional processes for each of the dependent parameters. If these probabilistic dependencies are not significantly strong then the situation can be simplified further by assuming independence between parameter processes.

The first step in developing a geologic prediction model is to select an appropriate probabilistic

model for $\underline{\mathbf{X}}(t)$ and to estimate its distribution parameters. However, this is not sufficient for describing the actual geologic conditions of a project. Tunneling decisions depend on the particular, *location-specific* sequence of geologic parameter states along the tunnel alignment, which, even though unknown, are already determined by nature. Knowing the distribution parameters for the geologic parameter *rock type*, for example, is sufficient for making general statements like: “the probability that a *granite* section follows a *schist* section is 0.6”, or “the average persistence of *granite* sections is 500 ft (152 m)”. Clearly, a much more useful prediction can be produced by introducing location-specific evidence to update the above predictions to: “the probability that a *granite* section follows a *schist* section 300 ft (91 m) away from the tunnel portal is 0.9”, or “the expected length of a *granite* section 300 ft (91 m) from the tunnel portal is 180 ft (55 m)”.

In general, the development of a geologic prediction model involves the following steps: (i) select a probabilistic model for the random process $\underline{\mathbf{X}}(t)$, (ii) estimate the distribution parameters of $\underline{\mathbf{X}}(t)$ in order to describe the overall (i.e. non location-specific) geologic variability in the project’s vicinity, and (iii) update this description using location-specific information in order to predict the true geologic conditions at any particular point along the project.

Modeling Assumptions

The geologic prediction model presented in this paper is based on the discrete-state, continuous-space Markov process. Its basic assumptions can be summarized as follows (Ioannou 1984):

- It is possible to define a set of geologic parameters (such as rock type, joint density, faulting, joint appearance, degree of weathering, permeability, etc.) which—for all practical design and construction purposes—provide a complete description of a project’s geologic conditions. Each of these parameters is associated with an enumerable set of feasible states (or values). Any number of feasible states can be assigned to each parameter in order to approximate continuous variables (such as joint density) to any degree of accuracy.
- In the absence of location-specific information (e.g. information from boreholes) linking certain parameter states with particular locations, each of the parameters describing the geology undergoes state transitions along the tunnel alignment according to the laws of the discrete-

state, continuous-space Markov process. The distribution parameters defining the Markov process for each geologic parameter are estimated based on general geologic information.

- Location-specific observations on parameter states obtained from exploration programs are used to update the geologic parameter state predictions of the underlying Markov processes for each parameter according to Bayes theorem.

In order to simplify the following mathematical presentation only one geologic parameter $X(t)$ (e.g. *rock type*) will be considered. Such a separate description of geologic parameters is sufficient if the components of $\underline{X}(t)$ are mutually independent. The model, however, can readily be extended to the case of dependent parameters using the conditional expansion (2).

Elements of the Markov Process

In its simplest form and within homogeneous regions the model treats each geologic parameter as an independent discrete-state, continuous-space Markov process. A Markov process is defined as a random process with single-step memory, where the state probabilities for the process $X(t_{i+1})$ depend only on the known previous parameter state closest to location t_{i+1} . The Markovian assumption of single step memory can be expressed more clearly in mathematical terms, as:

$$P[X(t_{i+1}) = x_{i+1} | X(t_i) = x_i, X(t_{i-1}) = x_{i-1}, \dots] = P[X(t_{i+1}) = x_{i+1} | X(t_i) = x_i] \quad (3)$$

where x_i, x_{i-1}, \dots are the outcomes of the random variables $X(t_i), X(t_{i-1}), \dots$ respectively, and $t_{i+1} > t_i > t_{i-1} > \dots$ are the corresponding locations along the tunnel alignment as measured from an arbitrary origin. Typically the origin ($t = 0$) is chosen to coincide with the beginning of the tunnel.

The continuous state, discrete state Markov process for a geologic parameter $X(t)$ with n states is completely described by the *transition intensity matrix* \underline{A} :

$$\underline{A} = [a_{ij}] \quad i, j = 1, 2, \dots, n \quad (4)$$

where

$$a_{ij} = \begin{cases} -c_i & i = j \\ c_i P_{ij} & i \neq j \end{cases} \quad (5)$$

The parameters P_{ij} are the *transition probabilities* and c_i are the *transition intensity coefficients*. A transition probability P_{ij} is defined as:

$$P_{ij} = P[\text{The next state of } X \text{ will be } j \text{ given the present state is } i] \quad (6)$$

For example, a transition probability for the geologic parameter *rock type* would be: “the probability of the next rock type being *granite* given that the current rock type is *quartzite*”.

The transition intensity coefficient c_i is related to the *extent* H_i of state i . The extent H_i represents the length for which parameter $X(t)$ (e.g. *rock type*) remains within a particular state i (e.g. *granite*)— from the point $X(t)$ makes a transition into state i to the point where a transition out of i occurs (Fig. 1). It can be easily proven that under the Markovian assumption of single-step memory the extent H_i follows an exponential distribution with the following PDF and CDF:

$$f_{H_i}(h) = c_i e^{-c_i h} \quad (7)$$

$$F_{H_i}(h) = 1 - e^{-c_i h} \quad (8)$$

The interval transition coefficient c_i equals the inverse of the expected value of the extent H_i . For example, if the expected length of a *granite* segment along the tunnel alignment is estimated to be $1000ft$ ($305m$), then the interval transition coefficient for the state *granite* of the geologic parameter *rock type* is $0.001ft^{-1}$ ($0.0033m^{-1}$).

Parameter Estimation

The estimation of the parameters P_{ij} and c_i can be accomplished by either statistical procedures or direct subjective assessment. The applicability of statistical methods is obviously dependent on the form and amount of available data. For example, if geological maps of the project area are available, then the parameters c_i can be estimated by computing the inverse of the average extent of each state, and P_{ij} can be estimated by drawing a set of lines parallel to the tunnel alignment— spaced far apart to avoid a strong dependence between them—and computing the ratio

$$P_{ij} = \frac{\text{Number of transitions from state } i \text{ to state } j}{\text{Total number of transitions out of state } i} \quad (9)$$

The Markov process is assumed to be *homogeneous* within regions with the same geologic history and thus the parameters P_{ij} and c_i are considered constants independent of location t .

If the project extends into several homogeneous regions then these parameters must be assessed separately for each case. A complete discussion concerning the applicability, the relative strengths and the ways these methods are used can be found in (Chan 1981).

Basic Multi-transition Probabilities

The probabilistic behavior of a Markov process $X(t)$ over multiple transition intervals is defined by the matrix of *interval transition probabilities* $\underline{\mathbf{V}}(t_0, t)$:

$$\underline{\mathbf{V}}(t_0, t) = [v_{ij}(t_0, t)] \quad i, j = 1, 2, \dots, n \quad (10)$$

The interval transition probability,

$$v_{ij}(t_0, t) = v_{ij}(t - t_0) = P[X(t) = j | X(t_0) = i] \quad (11)$$

is the probability that the process will be in state j at location t , given that it occupies state i at the current location t_0 ($t \geq t_0$).

The interval transition probabilities $\underline{\mathbf{V}}(u)$ of a Markov process satisfy the Kolmogorov differential equation:

$$\frac{d\underline{\mathbf{V}}(u)}{du} = \underline{\mathbf{V}}(u)\underline{\mathbf{A}} \quad (12)$$

and are given by the general solution:

$$\begin{aligned} \underline{\mathbf{V}}(u) &= e^{u\underline{\mathbf{A}}} \\ &= \underline{\mathbf{I}} + u\underline{\mathbf{A}} + (1/2!)u^2\underline{\mathbf{A}}^2 + \dots + (1/m!)u^m\underline{\mathbf{A}}^m + \dots \end{aligned} \quad (13)$$

where $\underline{\mathbf{I}}$ is the identity unit matrix of size n and $u = t - t_0$. A closed form expression for $\underline{\mathbf{V}}(u)$ can be constructed by using the spectral resolution of $\underline{\mathbf{A}}$ (Cox 1965), or by using exponential transforms (Howard 1960, 1971).

The probability that a Markov process $X(t)$ will occupy a particular state j at location t , irrespective of the current state i at t_0 , is given by the row vector of *marginal state probabilities* $\underline{\mathbf{S}}(t)$:

$$\underline{\mathbf{S}}(t) = [s_j(t)] \quad j = 1, 2, \dots, n \quad (14)$$

where,

$$s_j(t) = P[X(t) = j] \quad (15)$$

These probabilities can be computed from:

$$\begin{aligned}\underline{\mathbf{S}}(t) &= \underline{\mathbf{S}}(t_0)\underline{\mathbf{V}}(t_0, t) \\ &= \underline{\mathbf{S}}(t_0)e^{(t-t_0)\underline{\mathbf{A}}}\end{aligned}\quad (16)$$

where $\underline{\mathbf{S}}(t_0)$ is the row vector of marginal state probabilities at the current location t_0 . As the interval $t - t_0$ increases, the effect of the present state on the probabilities of future states at location t becomes smaller and smaller. When $t - t_0$ approaches infinity, the probability of finding a certain state j at location t becomes a limiting constant and is independent of the present state i . The corresponding vector of state probabilities is called the vector of *limiting state probabilities* $\underline{\mathbf{S}}^*$ and is given by the solution of the following homogeneous system of simultaneous linear equations:

$$\underline{\mathbf{S}}^*\underline{\mathbf{A}} = 0 \quad (17)$$

with the normalizing condition

$$s_1^* + s_2^* + \dots + s_n^* = 1 \quad (18)$$

The physical significance of s_j^* is that it is the relative percentage of the occurrence of state j . If state j (e.g. *granite*) of a parameter $X(t)$ (e.g. *rock type*) occurs 70% of the time in a certain region, then $s_j^* = 0.7$. For a tunnel of length l in such a region, the expected total length of granite is $s_j^*l = 0.7l$. The limiting state probabilities provide the initial prior distribution used to update the parameter state probabilities when observations are made available.

Updating State Predictions

The estimation of the Markov process parameters P_{ij} and c_i for each geologic parameter $X(t)$ is based on general information, i.e. information that is not location-specific. Location-specific observations are used to predict geologic parameter states as they exist at the site by updating the original interval transition probabilities $\underline{\mathbf{V}}(t_0, t)$ and state probabilities $\underline{\mathbf{S}}(t)$ of the underlying Markov process. The estimates of geologic parameter state probabilities at the observation locations are thus strongly dependent on location-specific information, whereas the prediction of state probabilities between or away from observation locations depend on several factors, such as the distance from observation locations, the state probabilities at these locations and the updated interval transition probabilities of the basic process.

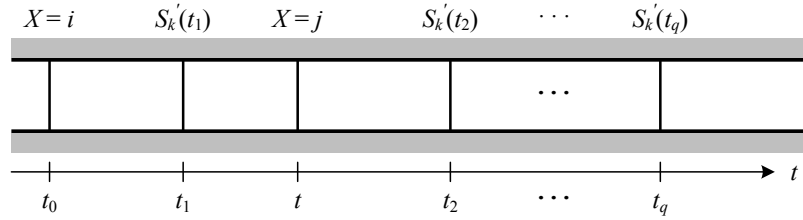


Figure 2: —Updating the tunnel profile based on q imperfect observations

The *posterior* state probabilities of the geologic parameter states at the observation locations can be determined using one of the following two approaches:

1. Subjective assessment by an expert geologist.
2. Bayesian updating based on the reliability of the method providing the observation.

The procedure for determining the updated state probabilities *at any other point* along the tunnel alignment is independent of the method used for obtaining the posterior state probabilities at the observation locations. Since this procedure is also the basis for the Bayesian approach for determining the posterior probabilities at the observation locations it is presented first.

Updating State Probabilities Between Observations

In general, a geologic exploration program provides a set of q non-deterministic (i.e. imperfect) observations $Y(t_b)$, $b = 1, 2, \dots, q$, at locations $t_1 < t_2 < \dots < t_q$ along the tunnel alignment (Fig. 2). In this section, the *posterior* geologic parameter state probabilities at these locations:

$$s'_k(t_r) = P[X(t_r) = k | \bigcap_{b=1}^q Y(t_b)] \quad k = 1, 2, \dots, n; \quad r = 1, 2, \dots, q \quad (19)$$

are assumed to be known. Under this assumption the fundamental problem is to determine the posterior state probabilities $s'_j(t)$ at any *other* point t along the tunnel. In general, the solution to this problem is a two-step process: (i) determine the posterior interval transition probabilities $v'_{ij}(t_0, t)$ from an arbitrary reference point t_0 ($t_0 \leq t_1$) to any other point t , given the posterior state probabilities at the observation locations $s'_k(t_r)$, and (ii) compute the posterior state probabilities $s'_j(t)$ using equation (16).

The updated $v'_{ij}(t_0, t)$ are given by the following three expressions depending on the value of t :

$$v'_{ij}(t_0, t) = \begin{cases} \sum_{k=1}^n s'_k(t_1) \frac{v_{ij}(t-t_0)v_{jk}(t_1-t)}{v_{ik}(t_1-t_0)} & t_0 < t < t_1 \\ \sum_{m=1}^n s'_m(t_{r-1}) \sum_{k=1}^n s'_k(t_r) \frac{v_{mj}(t-t_{r-1})v_{jk}(t_r-t)}{v_{mk}(t_r-t_{r-1})} & t_{r-1} < t < t_r \\ \sum_{k=1}^n s'_k(t_q) v_{kj}(t-t_q) & t_q < t \end{cases} \quad (20)$$

It is interesting to notice that, because of the assumption of single-step memory, the updated interval transition probabilities $v'_{ij}(t_0, t)$ are only a function of the non-deterministic observations immediately preceding and following point t . Furthermore, the posterior $v'_{ij}(t_0, t)$ do not depend on i , the state at the reference location t_0 and thus they are equivalent, by definition, to the posterior state probabilities $s'_j(t)$. As a result, the application of equation (16) in step (ii) above becomes unnecessary.

The only exception occurs when $t_0 < t < t_1$. Since this situation is of interest only for points t between the beginning of the tunnel and the first available observation, it is convenient to assign the arbitrary location t_0 to be the beginning of the tunnel. The procedure for determining the posterior state probabilities $s'_k(t_0)$ at t_0 is the same as for the observation locations—either direct assessment, or Bayesian updating—as explained below. Given these probabilities, the $v'_{ij}(t_0, t)$ are independent of state i and thus it is possible to set:

$$s'_j(t) = v'_{ij}(t_0, t) \quad i, j = 1, 2, \dots, n \quad (21)$$

Equations (20) can then be used to construct a probabilistic profile for each geologic parameter by computing the posterior state probabilities at any point along the tunnel alignment.

State Prediction at Observation Points.

The above expressions for predicting geologic parameter states *between* the direct observations provided by an exploration program are based on the assumption that the posterior state probabilities at the q observation locations are already available. These probabilities can be determined

either through direct subjective encoding or by assessing the reliability of the observation methods and applying Bayes theorem.

In theory both methods are equally valid (Ioannou 1984). In practice, the subjective encoding approach is more straightforward because it produces all the state probabilities at the observation locations directly, and does not require any computations. However, it requires a considerable effort in encoding expert human judgment (Chan 1981; Spetzler 1972). Furthermore, these probabilities must be reassessed every time a new observation is made available. Hence, this approach cannot be easily implemented, much less automated, when one wishes to continuously update the geologic profile by considering observations in series, one at a time. The method is best suited to situations where all observations are already available and can thus be considered simultaneously.

The Bayesian updating approach bypasses these limitations by using a pre-assessed reliability matrix for each of the observation methods. Even though the resulting procedure requires extensive calculations, it can be readily automated (Ioannou 1984).

The Likelihood (Reliability) Matrix.

The likelihood (or reliability) matrix of a *particular* exploration method is defined as:

$$L_{jk}(t_b) = P[Y(t_b) = k | X(t_b) = j] \quad j = 1, 2, \dots, n; \quad k = 1, 2, \dots, m \quad (22)$$

where,

n is the number of possible states for the geologic parameter $X(t)$.

m is the number of possible states for the observation $Y(t_b)$.

t_b is the location of the observation $Y(t_b)$.

$Y(t_b)$ is the observation at location t_b describing the state of the geologic parameter $X(t_b)$.

$X(t_b)$ is the true state of the geologic parameter at location t_b .

$L_{jk}(t_b)$ is the likelihood of observing state k at location t_b , given that the true parameter state is j .

In general, the number of observation states m does not have to equal the number of geologic parameter states n . Obviously the case where $m \geq n$ implies a more accurate observation method than if $m < n$. For the ease of notation in the above definition, the likelihood matrix $L_{jk}(t_b)$ is not associated explicitly with a particular geologic parameter or exploration method. In practice, however, a different likelihood matrix must be defined for every particular combination of exploration method and geologic parameter. The likelihood matrix is also assumed to be a function of location t_b since the reliability of an observation method may also depend on the general geologic conditions in the project area and not only on the parameter being observed. This captures the effect of other parameters on the reliability of the observations and thus models the general problem.

The One-Step Bayesian Updating Approach.

The updated state probabilities $s'_i(t_r)$ at each of the observation locations given *all* observations $Y(t_b)$:

$$s'_i(t_r) = P[X(t_r) = i | \bigcap_{b=1}^q Y(t_b) = k_b] \quad i = 1, 2, \dots, n; \quad r = 1, 2, \dots, q \quad (23)$$

can be directly computed by applying the Total Probability and Bayes theorems as follows:

$$s'_i(t_r) = \frac{\left\{ \sum_{j_1=1}^n \cdots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \cdots \sum_{j_q=1}^n [L_{j_1 k_1}(t_1) \cdots L_{i k_r}(t_r) \cdots L_{j_q k_q}(t_q) s_{j_1}(t_1) v_{j_1 j_2}(t_2 - t_1) \cdots v_{j_{r-1} i}(t_r - t_{r-1}) v_{i j_{r+1}}(t_{r+1} - t_r) \cdots v_{j_{q-1} j_q}(t_q - t_{q-1})] \right\}}{\sum_{j_1=1}^n \cdots \sum_{j_q=1}^n [L_{j_1 k_1}(t_1) \cdots L_{j_q k_q}(t_q) s_{j_1}(t_1) v_{j_1 j_2}(t_2 - t_1) \cdots v_{j_{q-1} j_q}(t_q - t_{q-1})]} \quad (24)$$

The numerator of this expression contains $q - 1$ summations (one less than the denominator) of the products enclosed in brackets. The interval transition probabilities $v_{jk}(t_w - t_r)$ on the right-hand side are determined according to the basic Markov process for all combinations of observation locations t_r, t_w . The state probabilities $s_j(t_1)$ at the first observation location are set equal to the limiting state probabilities s_j^* .

There are two major practical difficulties in implementing and using equation (24). The first is that the number of summations required for determining the numerator and the denominator depends on q and is thus variable. In programming terms this task is either impossible or extremely

memory consuming, depending on the computer language. The second problem is that the above expression describes a *one-step* solution which must be repeated entirely whenever a new observation becomes available. This makes the procedure computationally inefficient, especially for simulation purposes. The shortcomings of the direct method can be alleviated by the following alternative formulation which is also based on Bayes theorem.

The Iterative Bayesian Updating Approach.

Instead of considering all observations simultaneously, it is more expedient to update the state probabilities at each observation location by considering observations serially, one at a time. As explained earlier, the number of locations at which this updating needs to be performed is either q or $q + 1$, depending on whether an observation at t_0 (the beginning of the tunnel) does, or does not exist.

The basis for this approach is the probability that the true parameter state at an existing (or future) observation location t_r is $X(t_r) = i$, given that a single observation $Y(t_b) = k$ has been made at location t_b :

$$s'_i(t_r) = P[X(t_r) = i | Y(t_b) = k] \quad i = 1, 2, \dots, n; \quad r = 1, 2, \dots, q \quad (25)$$

These probabilities can be determined by using Bayes theorem as follows:

$$s'_i(t_r) = \begin{cases} \frac{\sum_{j=1}^n L_{jk}(t_b) v_{ij}(t_r, t_b) s_j(t_b)}{\sum_{j=1}^n L_{jk}(t_b) s_j(t_b)} & t_r < t_b \\ \frac{\sum_{j=1}^n L_{jk}(t_b) v_{ji}(t_b, t_r) s_j(t_b)}{\sum_{j=1}^n L_{jk}(t_b) s_j(t_b)} & t_b < t_r \\ \frac{L_{ik}(t_b) s_i(t_b)}{\sum_{j=1}^n L_{jk}(t_b) s_j(t_b)} & t_r = t_b \end{cases} \quad (26)$$

The above expressions for updating the state probabilities $s'_i(t_r)$ at all observation locations t_r are *iterative*. The right hand side refers to probabilities that are *prior* with respect to the observation currently being considered, but are *posterior* with respect to the observations considered

previously. Assuming a total of q observations, the state probabilities at all q (or $q + 1$) locations must be updated q times—once for each observation.

The interval transition probabilities $v_{ij}(t_r, t_b)$ on the right hand side must also be updated in each cycle since they are needed for updating the parameter state probabilities based on the next observation. Furthermore, the interval transition probabilities must now be written as $v_{ij}(t_r, t_b)$ as opposed to $v_{ij}(t_b - t_r)$. This change in notation is necessary because the posterior probabilities do not simply depend on the relative distance between observation locations (as is the case for the original Markov process) but also depend on the absolute observation locations.

The probability that the true geologic parameter state at the observation location t_w is state j , given that the true state at location t_r is i and that state k has been observed at location t_b , is given by the updated interval transition probability:

$$v'_{ij}(t_r, t_w) = P[X(t_w) = j | X(t_r) = i, Y(t_b) = k] \quad i, j = 1, 2, \dots, n; \quad r, w = 1, 2, \dots, q \quad (27)$$

where $t_r < t_w$ according to the assumed directionality of the Markov process. Mathematically these probabilities are given by:

$$v'_{ij}(t_r, t_w) = \begin{cases} v_{ij}(t_r, t_w) & t_b < t_r < t_w \\ \frac{\sum_{h=1}^n L_{hk}(t_b) v_{hj}(t_b, t_w) v_{ih}(t_r, t_b)}{\sum_{h=1}^n L_{hk}(t_b) v_{ih}(t_r, t_b)} & t_r < t_b < t_w \\ \frac{\sum_{h=1}^n L_{hk}(t_b) v_{jh}(t_w, t_b) v_{ij}(t_r, t_w)}{\sum_{h=1}^n L_{hk}(t_b) v_{ih}(t_r, t_b)} & t_r < t_w < t_b \end{cases} \quad (28)$$

The interval transition probabilities $v_{ij}(t_r, t_b)$ on the right hand side of these equations are *prior* to the observation $Y(t_b)$ and *posterior* to all other observations already considered and are thus computed in the previous iteration cycle. The same prior $v_{ij}(t_r, t_b)$ are also used in equations (26).

The procedure for updating the state probabilities at all q (or $q + 1$) observation locations begins by assuming that no observations are available. In this case, the prior $v_{ij}(t_r, t_b)$ and $s_i(t_r)$

are given by the interval transition probabilities and limiting state probabilities of the underlying Markov process. One of the q observations is then assumed to be available and is added to the set of known observations. The posterior $s'_i(t_r)$ and $v'_{ij}(t_r, t_b)$ are then determined using equations (26) and (28). The resulting $s'_i(t_r)$ and $v'_{ij}(t_r, t_b)$ become the new priors for considering the next available observation and the process is repeated until all q observations are considered. The final $s'_i(t_r)$ and $v'_{ij}(t_r, t_b)$ computed through this iterative procedure are identical to those obtained through the one-step approach using equation (24).

The final posterior state probabilities $s'_i(t_r)$ at the q (or $q + 1$) observation locations become the input to equations (20) to compute the updated geologic parameter state probabilities at any other point along the tunnel alignment.

The Geologic Prediction Model.

In order to put the above equations in perspective it is necessary to discuss the input requirements of the geologic prediction model and to provide an outline of the calculation procedures for determining the final geologic parameter profile.

The input requirements of the model are:

1. The definition of the N geologic parameters and the definition of the n states for each parameter.
2. Estimates of the transition probabilities P_{ij} , and the transition intensity coefficients c_i for each geologic parameter and for each possible state. These parameters can be estimated by (a) subjective encoding using a single or multiple experts, (b) statistical evidence from geologic area maps, or (c) a combination of the two.
3. The definitions of the m observation states and the likelihood matrices L_{jk} ($j = 1, \dots, n$; $k = 1, \dots, m$) for each combination of observation method and geologic parameter. The number of observation states and the reliability of observation methods is estimated from performance data in other projects.
4. The number of available observations q for each geologic parameter, the corresponding observation locations t_b and the observation states $Y(t_b) = k_b$.

Given the above data the sequence of operations for geologic prediction proceeds as follows:

1. The P_{ij} and c_i for each parameter are used to construct the transition matrix $\underline{\mathbf{A}}$ which is then used to determine the interval transition probabilities $\underline{\mathbf{V}}(t_0, t)$, state probabilities $\underline{\mathbf{S}}(t)$ and limiting state probabilities $\underline{\mathbf{S}}^*$ for the corresponding Markov processes using equations (13) (16) (17) (18).
2. One of the q observations $Y(t_b)$ is added to the list of available observations, and:
 - (a) The updated state probabilities $s'_i(t_r)$ at all observation locations t_r (and at t_0) are computed using equations (26), and
 - (b) The updated interval transition probabilities $v'_{ij}(t_r, t_w)$ for all possible combinations of observation locations t_r and t_w are determined using equations (28).

Steps (a) and (b) are repeated in sequence for all q observations and for all geologic parameters. The initial prior probabilities at all observation locations are provided by the interval transition probabilities $\underline{\mathbf{V}}(t_0, t)$ and the limiting state probabilities $\underline{\mathbf{S}}^*$ computed in step (1).

3. Equations (22) are used to determine the updated state probabilities $s'_j(t)$ by computing the equivalent posterior interval transition probabilities $v'_{ij}(t_0, t)$ for all the required points between direct observations based on the updated state probabilities $s'_j(t_r)$ at the observation locations. The set of posterior state probabilities at all points along the tunnel alignment forms the probabilistic geologic profile for the project.

Ground Classes

Even though the individual geologic parameter profiles established through the above procedures provide a complete description of project geology they are not in a form that can be used directly for decision making purposes. For problems such as estimating the cost of construction given a particular sequence of excavation-support methods, or determining the optimum sequence of excavation-support methods that minimizes the overall project cost, it is necessary to simultaneously consider the geologic profiles for all parameters in the form of a geologic vector $\underline{\mathbf{X}}(t)$. The number of possible vector states $\underline{\mathbf{x}}$ for the random process $\underline{\mathbf{X}}(t)$, however, is often too large to be

useful in practice. For example, if there are 5 geologic parameters ($N = 5$) each having 4 possible states ($n = 4$), then the number of possible vector states \underline{x} would be $4^5 = 1024$. Since it is practically impossible to assign a different construction alternative to each of these states, the correspondence of excavation-support methods to particular combinations of geologic vectors is typically accomplished through the definition of a set of *ground classes* (Einstein 1983; Ioannou 1984). The concept of ground classes is based on the following:

- For any project there exists a finite set of alternative excavation-support methods CM_i , $i = 1, 2, \dots, g$, of which at least one (i.e. CM_g) is adequate for the construction of the tunnel under the worst possible geologic conditions \underline{x} . These methods can be arranged and indexed according to their cost and applicability so that it is technically feasible to use a more expensive method under all geologic conditions for which a less expensive method is adequate. As a result, the least expensive method CM_1 can be substituted by CM_2, \dots, CM_g and CM_2 can be substituted by CM_3, \dots, CM_g , whereas CM_g cannot be substituted by any other method.
- A ground class gc_i , $i = 1, 2, \dots, g$, is then defined as the set of geologic vector states \underline{x} for which the adoption of construction method CM_i is the most economical and feasible alternative. The number of ground classes is obviously equal to the number of excavation-support options considered.

The definition of ground classes reduces the dimensionality of the problem to the level required for making design and construction decisions. Instead of the vector random process $\underline{X}(t)$ with n^N possible states (assuming that n states have been defined for each geologic parameter), the geologic variability along the tunnel axis can thus be described by a (non-Markovian) scalar random process GC whose possible states gc_i are the g ground classes. The *ground class profile* of a tunnel is simply the set of ground class state probabilities at each point along the tunnel alignment. These probabilities are easily computed as the sum of the corresponding state probabilities of the geologic vector states \underline{x} that belong to each ground class. The methodology for determining the ground class profile is illustrated in the example application of the model.

The tunnel ground class profile is the final output of the geologic prediction model. This profile provides an efficient and sufficient description of geologic variability which can then be

used as direct input to design and construction decision support systems. Several such systems have already been developed, including:

- Probabilistic cost estimating of tunnel construction (Ioannou 1984).
- Heuristic selection of excavation-support sequence in non-adaptable tunnel construction (Ioannou 1984).
- Optimal selection of excavation-support sequence in adaptable tunneling using stochastic dynamic programming (Kim 1984).
- The economic evaluation of geologic exploration programs (Ioannou 1984, 1985).

An Example Application

This section presents an application of the proposed geologic prediction model using as an example the discharge water tunnel of the Seabrook Power Station (Ioannou 1984; Ashley 1981). This example illustrates the model's data input requirements and the form of the corresponding model output: the geologic parameter profiles and the associated ground class profile.

The actual Seabrook Power Station discharge tunnel is over $15,000ft$ ($4572m$) long. This example considers only the western portion from boreholes ADT-1 ($t = 0ft$) to ADT-42 ($t = 7662ft$ ($2335m$)) as shown in the estimated Rock Type Profile, Fig. 3 (Chan 1981). A detailed description of the project's regional geology can be found in (Rand 1974).

Four geologic parameters X_i are used to describe the tunnel characteristics: $X_1 = \text{Rock Type}$, $X_2 = \text{Joint Density (RQD)}$, $X_3 = \text{Degree of Weathering}$ and $X_4 = \text{Availability of Water}$. The number of discrete states n_i and their definitions for each geologic parameter are shown in Table 1.

Table 2 shows the transition intensity coefficients c_i and the transition probabilities P_{ij} for each geologic parameter. These parameters were directly encoded using a geologist's expert judgment and were based on available general information. Table 2, for example, indicates that the geologist's estimate for the probability that a *diorite* segment ($X_1 = 3$) is followed by a *metaquartzite* segment ($X_1 = 2$) is $P_{32} = 0.20$ and that the average extent of a *diorite* segment is estimated to be $1/c_3 = 1/0.00262 = 382ft$ ($116m$).

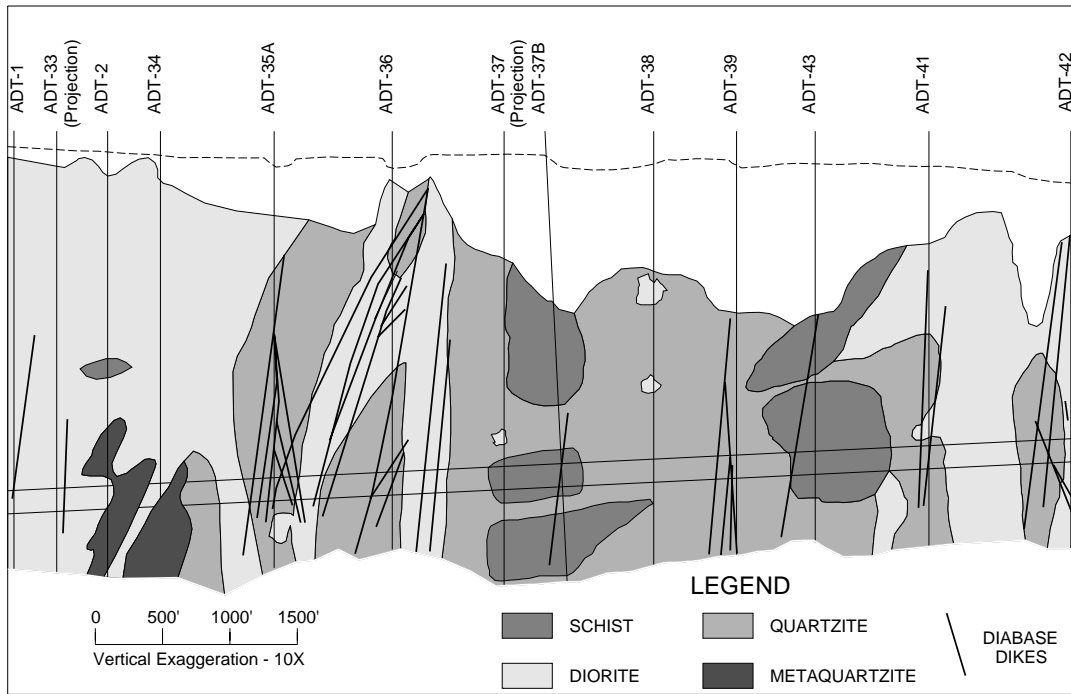


Figure 3: Rock Type Profile, Seabrook Power Station Discharge Tunnel

Table 1: —Geologic Parameter and State Definition

Geologic Parameters ($N = 4$)							
1. Rock Type X_1		2. Joint Density (RQD) X_2		3. Degree of Weathering X_3		4. Water Availability X_4	
i	State	i	State	i	State	i	State
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Schist	1	75-100%	1	Not Severe	1	Low
2	Metaquartzite	2	25-75%	2	Severe	2	Medium
3	Diorite	3	0-25%			3	High
4	Quartzite						
$n_1 = 4$		$n_2 = 3$		$n_3 = 2$		$n_4 = 3$	

Table 2: —Interval Transition Probabilities P_{ij} and Interval Transition Coefficients c_i

i	Rock Type X_1					Joint Density (RQD) X_2				Degree of Weathering X_3			Water Availability X_4			
	P_{ij}				c_i (a)	P_{ij}			c_i (a)	P_{ij}		c_i (a)	P_{ij}			c_i (a)
	$j = 1$	2	3	4		1	2	3		1	2		1	2	3	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	.00	.02	.23	.75	138	.00	.93	.07	233	.00	1.0	45	.00	.55	.45	82
2	.02	.00	.50	.48	822	.83	.00	.17	394	1.0	.00	162	.90	.00	.10	633
3	.02	.20	.00	.78	262	.50	.50	.00	188				.90	.10	.00	170
4	.23	.17	.60	.00	250											

(a) Units: $10^{-5} ft^{-1}$. Note: $3.28ft = 1m$

Although no frequency data were available for estimating the necessary Markov process parameters for this project, there were thirteen point observations ($q = 13$) obtained from boreholes along the tunnel axis. The observations $Y_i(t_b)$ for each geologic parameter X_i and their locations t_b are shown in Table 3. For the purposes of this example, the definitions of the observation states are identical to the definitions of the geologic parameter states (Table 1). For example, Table 3 shows that the borehole observations at location $t = 1945ft$ ($593m$) indicated that the rock was *quartzite* ($Y_1(1945) = 4$), moderately jointed with $RQD = 25 - 75\%$ ($Y_2(1945) = 2$), *not severely weathered* ($Y_3(1945) = 1$), with *high availability of water* ($Y_4(1945) = 3$).

The estimated likelihood (reliability) matrices $L_{ij}(t_b)$ for the borehole observations have been assumed independent of location t_b and are shown in Table 4. From this table, for example, it can be seen that the probabilities that a borehole core sample would indicate *low*, *moderate* or *high* joint density, given that the true joint density is *moderate*, are $P[Y_2 = 1|X_2 = 2] = 0.05$, $P[Y_2 = 2|X_2 = 2] = 0.85$, $P[Y_2 = 3|X_2 = 2] = 0.10$, respectively.

The above data constitute the basic input to the geologic prediction model. The probabilistic profiles for each geologic parameter were determined by computing the posterior geologic parameter state probabilities $s'_i(t)$ using the procedures outlined in the previous section. The resulting profiles for the geologic parameters *Rock Type* (X_1) and *Joint Density* (X_2) are shown in Figs. 4 and 5. The profiles for the other two geologic parameters (X_3 and X_4) are similar. Each of these profiles shows the updated state probabilities $s'_i(t)$ for the associated geologic parameter as

Table 3: —Borehole Locations and Observed States

<i>b</i>	Bore-hole	<i>t_b</i> (<i>ft</i>)	Observed States			
			<i>Y</i> ₁ (<i>t_b</i>)	<i>Y</i> ₂ (<i>t_b</i>)	<i>Y</i> ₃ (<i>t_b</i>)	<i>Y</i> ₄ (<i>t_b</i>)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1	0	3	2	1	1
2	33	341	3	1	1	1
3	2	717	3	1	1	1
4	34	1239	2	2	1	3
5	35A	1945	4	2	1	3
6	36	2788	4	3	2	1
7	37	3566	1	2	2	3
8	37B	4010	1	3	2	3
9	38	4659	4	3	2	1
10	39	5256	4	1	1	2
11	43	5785	1	1	1	1
12	41	6604	3	1	1	1
13	42	7662	4	1	1	1

Note: 3.28 *ft* = 1 *m*

Table 4: —Estimates for the Likelihood (Reliability) Matrices

<i>i</i>	$L_{ij} = P[Y_k = j X_k = i]$											
	Rock Type				Joint Density (RQD)			Degree of Weathering		Water Availability		
	<i>(k = 1)</i>				<i>(k = 2)</i>			<i>(k = 3)</i>		<i>(k = 4)</i>		
(1)	<i>j = 1</i>	2	3	4	<i>j = 1</i>	2	3	<i>j = 1</i>	2	<i>j = 1</i>	2	3
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	0.85	0.05	0.05	0.05	0.90	0.05	0.05	0.95	0.05	0.95	0.05	0.00
2	0.05	0.85	0.05	0.05	0.05	0.85	0.10	0.05	0.95	0.05	0.90	0.05
3	0.05	0.05	0.85	0.05	0.00	0.05	0.95			0.00	0.05	0.95
4	0.05	0.05	0.05	0.85								

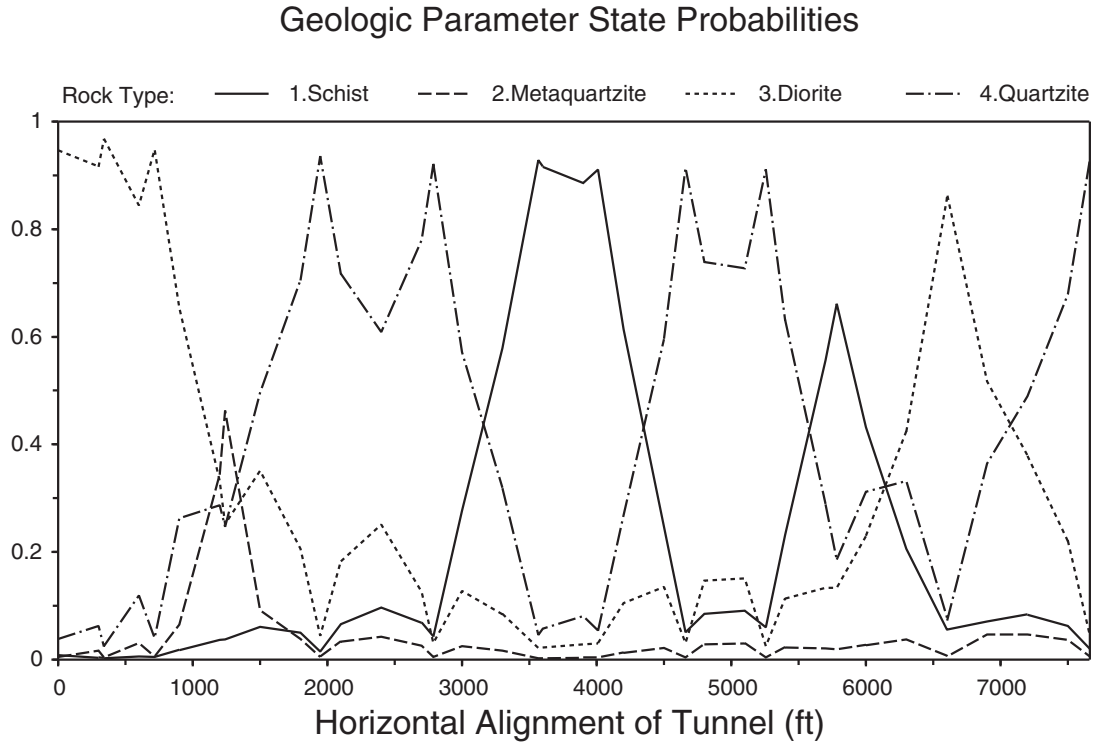


Figure 4: Probabilistic Geologic Parameter Profile—Rock Type

a function of location t along the tunnel axis. Fig. 4, for example, shows that the updated *Rock Type* state probabilities at location $t = 2400\text{ft}$ (732m), for the states *schist*, *metaquartzite*, *diorite* and *quartzite* are $P[X_1 = 1] = 0.097$, $P[X_1 = 2] = 0.043$, $P[X_1 = 3] = 0.251$, and $P[X_1 = 4] = 0.609$, respectively.

Five tunnel excavation-support alternatives CM_i , have been identified as suitable for the construction of this project ($g = 5$). These alternatives have been ordered according to their cost and technical feasibility as shown in Table 5. This table also shows the definitions of the corresponding five ground classes gc_i . Each row lists a set of states for each geologic parameter $[X_1, X_2, X_3, X_4]$. The possible combinations of geologic parameter states $[x_1, x_2, x_3, x_4]$ in each row define geologic vectors \underline{x} that belong to the corresponding ground class. For example, ground class gc_2 is defined by the two vectors $[4, 1, 1, 1]$ and $[4, 1, 1, 2]$. The number of possible geologic vectors \underline{x} for this example is $4 \times 3 \times 2 \times 3 = 72$, 6 of which belong to gc_1 , 2 belong to gc_2 , 8 belong to gc_3 , 20 belong to gc_4 , and 36 belong to gc_5 .

The ground class profile for the Seabrook tunnel is shown in Fig. 6. This profile shows the ground class state probabilities as a function of location along the tunnel axis. The state probabil-

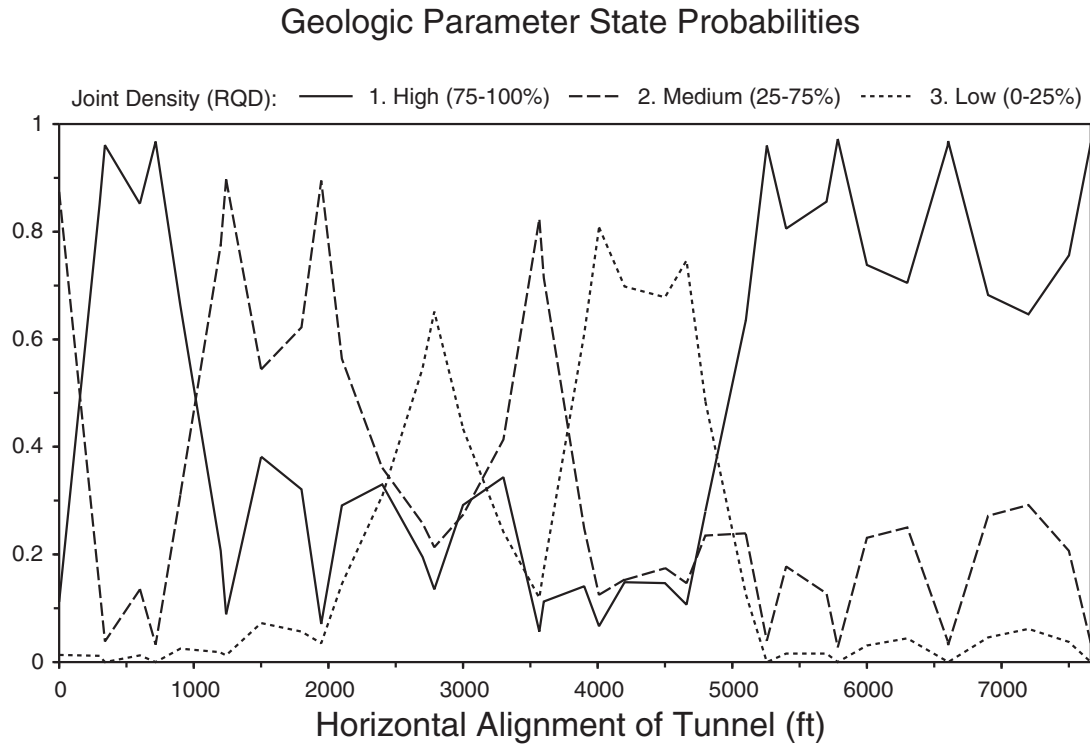


Figure 5: Probabilistic Geologic Parameter Profile—Joint Density (RQD)

Table 5: —Excavation-Support Methods and Ground Classes

<i>i</i> (1)	Excavation-Support Method CM_i (2)	Ground Class GC (3)	Geologic Parameter States			
			X_1 (4)	X_2 (5)	X_3 (6)	X_4 (7)
1	Full face drill and blast No support	gc_1	1, 2, 3	1	1	1, 2
2	Full face drill and blast Conventional steel sets Amount of support: medium	gc_2	4	1	1	1, 2
3	Heading & bench drill and blast Conventional steel sets Amount of support: medium	gc_3	1, 2, 3, 4	2	1	1, 2
4	Heading & bench drill and blast Conventional steel sets Amount of support: large	gc_4	1, 2, 3, 4 1, 2, 3, 4	1, 2, 3 3	1 1	3 1, 2
5	Multiple drift drill and blast Conventional steel sets Amount of support: large	gc_5	1, 2, 3, 4	1, 2, 3	2	1, 2, 3

ities for each ground class are computed by summing the state probabilities of the corresponding geologic vectors \underline{x} . In general, the state probabilities for each geologic vector \underline{x} are calculated using the conditional expansion (2). For the purposes of this example the four geologic parameters have been assumed independent and as a result, expansion (2) leads to the simple expression:

$$P[\underline{\mathbf{X}}(t) = \underline{\mathbf{x}}] = P[X_1(t) = x_1]P[X_2(t) = x_2]P[X_3(t) = x_3]P[X_4(t) = x_4] \quad (29)$$

Fig. 6, for example, shows that the state probability for $gc_2 = \{[4, 1, 1, 1], [4, 1, 1, 2]\}$ at location $t = 2400ft (732m)$ is $P[GC(2400) = gc_2] = 0.081$. This probability can be computed as follows:

$$\begin{aligned} P[GC(2400) = gc_2] &= P[X_1(2400) = 4, X_2(2400) = 1, X_3(2400) = 1, X_4(2400) = 1] \\ &\quad + P[X_1(2400) = 4, X_2(2400) = 1, X_3(2400) = 1, X_4(2400) = 2] \\ &= P[X_1(2400) = 4]P[X_2(2400) = 1]P[X_3(2400) = 1]P[X_4(2400) = 1] \\ &\quad + P[X_1(2400) = 4]P[X_2(2400) = 1]P[X_3(2400) = 1]P[X_4(2400) = 2] \\ &= (0.609)(0.330)(.611)(0.608) + (0.609)(0.330)(.611)(0.055) \\ &= 0.081 \end{aligned} \quad (30)$$

Conclusion

As illustrated by the above example, the geologic prediction model is a means for explicitly structuring the information customarily used by engineers and geologists in assessing the geologic conditions of a tunneling project. The objectives of the model are (i) to formalize and expedite the prediction procedure using automation, (ii) to develop a geologic description that reflects the uncertainty of the information on which it is based, and (iii) to provide the necessary input to other decision support systems.

The model achieves these objectives to a satisfactory degree. Its final output, in the form of a ground class probabilistic profile, can be directly used in adaptable (using the New Austrian Tunneling Method) and conventional tunneling for both design and construction purposes. Related research efforts have capitalized on this capability to develop systems for the optimization of design and construction decisions (Ioannou 1984, 1985; Kim 1984). Furthermore, the model's updating

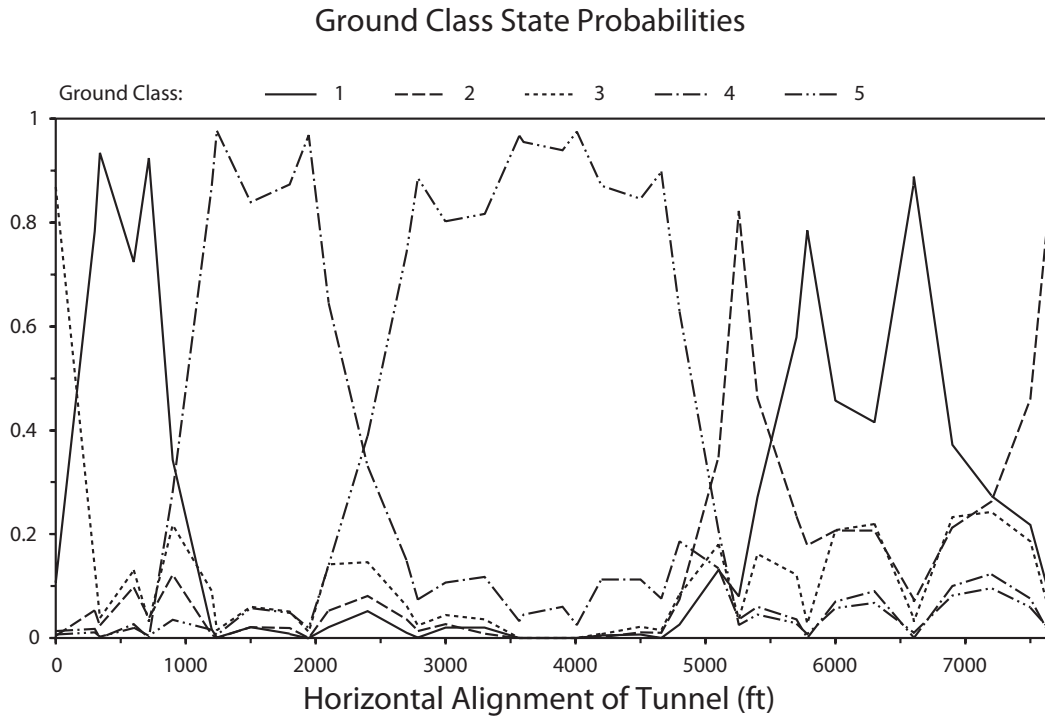


Figure 6: Probabilistic Ground Class Profile

procedure can be easily expanded to predict the results of future geologic exploration programs as a function of the reliability of the observation methods employed. The effectiveness of potential exploration strategies can then be evaluated by simulating future observations and updating the ground class profile (using the procedure outlined in this article). A decision support system for the evaluation of exploration programs is described in (Ioannou 1984, 1985).

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Appendix II. — Notation

The following symbols are used in this paper:

- $\underline{\mathbf{A}}$ = Transition matrix for the discrete-state, continuous-space Markov process $X(t)$.
- c_i = Transition intensity coefficient for state i of the Markov process $X(t)$.
- CM_i = The i th excavation-support method.
- g = The number of excavation-support methods (ground classes).
- GC = The random process describing the variability of ground classes.
- gc_i = The i th ground class.
- H_i = The extent of state i of the Markov process $X(t)$.
- $\underline{\mathbf{I}}$ = The identity unit matrix of size n .
- m = Number of discrete observation states for the geologic parameter $X(t)$.
- n = Number of discrete states for the geologic parameter $X(t)$.
- $L_{ij}(t_b)$ = Likelihood of the observation state j at location t_b , given that the geologic parameter state at t_b is i .
- N = Number of geologic parameters: dimension of the vector random process $\underline{\mathbf{X}}(t)$.
- P_{ij} = Transition probability from state i to state j of the Markov process $X(t)$.
- q = Number of the (non-deterministic) observations $Y(t_b)$.
- s_i^* = Limiting state probability for state i of the Markov process X .
- $s_i(t)$ = Marginal state probability for state i at location t , of the Markov process $X(t)$.
- $s'_i(t)$ = Updated marginal geologic parameter state probability for state i at location t , given q (non-deterministic) observations $Y(t_b)$.

$s'_i(t_r)$ = Updated marginal state probability for state i at t_r (one of the q observation locations), given some or all of the q (non-deterministic) observations $Y(t_b)$.

t = The distance of a point along the alignment of a tunnel as measured from an arbitrary origin t_0 (usually the portal or starting point of the tunnel).

t_b, t_r, t_w = The locations along the tunnel alignment of the q observations $Y(t_b)$.

$v_{ij}(t - t_0)$ = The interval transition probability from state i at location t_0 , to state j at location t , for the Markov process $X(t)$.

$v'_{ij}(t_r, t_w)$ = The updated interval transition probability from state i at location t_r , to state j at location t_w , given some or all of the q (non-deterministic) observations $Y(t_b)$.

$X(t)$ = A generic geologic parameter as a scalar random process dependent on location t .

$\underline{X}(t)$ = A set of geologic parameters as a vector random process dependent on location t .

$Y(t_b)$ = A random variable with m states representing the observation on parameter $X(t)$ at location t_b .