CENTER FOR CONSTRUCTION ENGINEERING AND MANAGEMENT

REPETITIVE SCHEDULING METHOD

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REPETITIVE SCHEDULING METHOD

INTRODUCTION

Construction contractors often encounter projects that contain several identical or similar units, such as floors in multistory buildings, houses in housing developments, meters in pipelines, or stations in highways. These multi-unit projects are characterized by repeating activities, which in most instances arise from the subdivision of a generalized activity into specific activities associated with particular units. For example, a *Paint Walls* activity for a multistory building may be broken into *Paint First Floor Walls, Paint Second Floor Walls*, and so forth, where each floor is a significant unit of the overall project.

Activities that repeat from unit to unit create a very important need for a construction schedule that facilitates the uninterrupted flow of resources (i.e., work crews) from one unit to the next, since it is often this requirement that establishes activity starting times and determines the overall project duration. Hence, uninterrupted resource utilization becomes an extremely important issue.

The scheduling problem posed by multi-unit projects with repeating activities is akin to the minimization of the project duration subject to resource continuity constraints as well as technical precedence constraints. The uninterrupted deployment of resources is not a problem addressed by the Critical Path Method (CPM), nor by its resource-oriented extensions, such as time-cost tradeoff, limited resource allocation, and resource leveling.

However, this need for the uninterrupted utilization of resources from an activity in one unit to the same (repeating) activity in the next unit is explicitly recognized by several scheduling methodologies that have been available for many years and have been called by a number of different names. For projects with discrete units, such as floors, houses, apartments, stores, or offices, names that have been used include: *Line of Balance* (LOB) (O'Brien 1969, Carr and Meyer 1974, Halpin and Woodhead 1976, Harris and Evans 1977); *Construction Planning Technique* (CPT) (Peer 1974, Selinger 1980); *Vertical Production Method* (VPM) (O'Brien 1975, Barrie and Paulson 1978); *Time-Location Matrix Model* (Birrell 1980); *Time Space Scheduling Method* (Stradal and Cacha 1982); *Disturbance Scheduling* (Whitman and Irwig 1988); or HVLS: *Horizontal and Vertical Logic Scheduling for Multistory Projects* (Thabet and Beliveau 1994).

For highways, pipelines, tunnels, etc., where progress is measured in terms of horizontal length, the names used have included: *Time Versus Distance Diagrams* (Gorman 1972); *Linear Balance Charts* (Barrie and Paulson 1978); *Velocity Diagrams* (Dressler 1980); or *Linear Scheduling Method* (LSM) (Johnston 1981, Chrzanowski and Johnston 1986, Russell and Casselton 1988).

Although each of these methods was developed to meet its own particular objectives, all of them are essentially alike in that they schedule the work in the project by plotting the progress of repeating activities against time. Therefore, these methods can be integrated into one generalized and simplified model, the Repetitive Scheduling Method (RSM), that ensures uninterrupted resource utilization and is applicable to both vertical and horizontal construction (Harris and Ioannou 1998). Two significant concepts that emerge from the development of RSM are control points and the controlling sequence.

RSM is not a complicated technique. It is a simple and easily applied scheduling methodology that follows naturally from the concepts and relationships found in CPM precedence networks. Nevertheless, RSM has its limitations and there are project situations where it may be desirable to model parts of a project by CPM and other parts by RSM. Such an integrated project will be demonstrated in later sections of this work.

CPM MULTI-UNIT SCHEDULING

Multi-unit projects can be scheduled using commonly accepted CPM techniques, but continuous utilization of resources across repeating units cannot be assured when these CPM networks are used. This shortcoming is best illustrated by an example.

Figure 1 is a CPM network prepared for a project consisting of three repeating units of work. The solid lines linking the activities within each unit and linking similar activities from unit to unit represent the technical precedence constraints in the network; for example, Activities B1, C1, and A2 cannot be started until Activity A1 is completed. The dashed lines linking similar activities from unit to unit represent resource availability constraints; for example, Activity A2 cannot begin until the crew of carpenters from Activity A1 is available.

Note that Units 1 and 3 each have five activities, A through E, but Unit 2 does not contain a B activity. Unit 2 also differs in that the individual activity durations are not the same as in Units 1 and 3. These differences reflect the various amounts of work needed to complete the activities of the unit.

The solution of the network in Figure 1 results in a project duration of 18 days and a critical path that includes Activities A1, C1, C2, D2, D3, and E3. Typically, each unit in a repetitive network contains the same activities having the same durations, and the critical path passes through the network of activities in the first unit until an activity with a long duration is found. The path then passes through similar activities in successive units until the last unit in the sequence is reached, and continues through the last unit network until the final activity is completed. Had all three units in Figure 1 been alike, the path would have included Activities A1, C1, C2, C3, D3, and E3. The shift in the path to include Activity D2 and not C3 as expected is caused by the activity differences in Unit 2.

The links in this CPM network ensure that both technical precedence and resource availability requirements are met. However, resource continuity constraints cannot be represented directly in CPM networks, so the uninterrupted utilization of resources from unit to unit cannot be assured. The schedule shown in Figure 1 *does* provide for the continuous usage of the resource used by the C activities. Activity C1 begins on Day 3 and ends on Day 7, Activity C2 begins on Day 7 and ends on Day 10, and Activity C3 begins on Day 10 and ends on Day 14, so the use of the resource is uninterrupted from Day 3 to Day 14. (Notice that this continuous resource usage was neither required nor could have been anticipated).

For the D activities, the scheduled times do not provide continuous resource utilization. Activity D1 is scheduled to finish on Day 9, but the start of Activity D2 is not scheduled to begin until one day later (i.e., on Day 10). Therefore, there is a one day gap in the utilization of the resource needed for the D activities. Similarly, resource continuity is provided by the schedule of the A activities, but is not achieved for the B and E activities.

When uninterrupted utilization of resources is needed, activities having breaks in resource continuity can be rescheduled using their float times. For example, the one day Total Float for Activity D1 can be utilized, and D1 can be rescheduled to start on Day 8 and end on Day 10. In large projects with repeating activities, a complete activity-by-activity analysis and correction of the CPM network is required to ensure resource continuity, a process that is cumbersome and fraught with the possibility of error.

It may also be concluded from Figure 1 that CPM networks for projects with repeating units of work have a ladder-like appearance where each rung is a subnetwork that consists of the activities and precedence links for one unit. Because CPM diagrams show all the linkages between similar activities in successive units, the number of links and nodes will likely be large and the network will appear unnecessarily complicated.

RSM SCHEDULE REPRESENTATION

In contrast to the complex CPM network for scheduling multi-unit projects, an RSM schedule is presented graphically as an X-Y plot where one axis represents units, and the other time. The repetitive units may be assigned to either axis of the plot, the particular assignment being chosen for convenience and to clearly communicate the schedule information. For vertical construction projects, the repetitive units are usually discrete entities, such as houses, stores, apartments, or floors in high-rise construction, and work progress is measured in units completed. Hence, the units are typically shown along the Y-axis and time is shown along the X-axis. For horizontal construction projects, such as highways, pipelines, canals, tunnels, and so forth, work progress is measured in units of length and these units are shown along the X-axis to correlate with horizontal and vertical alignment charts, while time is shown along the Y-axis.

The repetitive units of the project must be arranged in some logical sequence along the chosen axis to define their pattern of repetition. This sequence may be accepted as a natural occurrence or may be established to suit some production need. For example, building floors must naturally be constructed one upon another, but houses in a development might be planned to follow in the order of their projected sale. Similarly, stations along a highway may follow in the natural numerical order from project start to project finish, or may be planned to recognize particular site or traffic conditions.

RSM ACTIVITY LOGIC

In addition to establishing the pattern by which repetitive units follow each other, it is necessary to identify the precedence constraints among the activities in each unit. To do so, a CPM precedence network is prepared for each typical repetitive unit, or if necessary, for each non-typical unit. These diagrams are similar to those shown for each unit in Figure 1.

The process to establish unit activity logic begins with the creation of a list of all the time consuming activities necessary for the completion of the project. Each activity in the list is given a name and identification symbol for easy reference, and the list is analyzed to determine the proper dependency relationships and to remove redundancies.

An examination of the activity list will most likely show groups of similar activities occurring again and again. For example, it may be observed that activities describing the construction of the first typical floor of a multistory building are repeated for several other succeeding floors. The collection of activities needed for each floor represents the details of a repetitive unit that is identified with that floor. The number of activities in the repetitive unit is not an important matter, because it is determined by the nature of the project. In some instances, the unit may contain only one activity.

Once all activities belonging to each repetitive unit have been identified, a logic diagram is prepared. This diagram can be either in the form of an arrow or a precedence network, but the precedence form is preferred. Each unit network should contain all production and want logic relationships among the activities. Because the main purpose of this diagram is to establish logical relationships among the activities, resource considerations within this unit can be temporarily ignored.

These steps to identify and logically relate the activities in a repetitive unit have been delineated here to serve as a reference procedure even though they are often performed simultaneously in practice. Actual projects may have complex relationships among their activities which may be difficult to properly detail without such a reference procedure.

Activity Logic Constraints

While the activities within a repetitive unit must be logically related, they also must be logically related from unit to unit according to the logical sequence pattern of the units as previously described. There are two types of constraints that control unit-to-unit logic in RSM diagrams; one is a technical precedence constraint and the other is a resource availability constraint. In the first instance, a particular work activity in the network of one unit must be followed by a similar work activity in the network of a succeeding unit to ensure that the flow of the technical work between the units is maintained. In the second case, the resource assigned to an activity in one unit also must be assigned to the similar activity in the succeeding unit to ensure that the resource required in the first unit is available when needed by the second unit. Note that this does not ensure that the resource between the two units will be used continuously.

RESOURCE CONSIDERATIONS

Every activity requires the application of resources for its performance, and all five basic resource types, material, equipment, labor, money, or time, or any combination of them, can be associated with it. In fact, most activities require that several resources be employed together; a piece of equipment needs an operator, for instance. RSM assumes that only the most significant resource is associated with an activity, and that all activities have been defined using this assumption.

It is also assumed that the same resource will be used for like activities in successive repeating units, so each activity's resource must be consistent from unit to unit. For example, if an activity in the first unit requires a crew of carpenters, that activity in each succeeding unit will require the same crew of carpenters.

Sometimes an activity needs several significant resources for its performance. This condition may be particularly evident when planning for equipment installations. The installation of heating, ventilating and air conditioning equipment in separate divisions of a warehouse may be taken as an example. The warehouse divisions are the repeating units and each piece of equipment requires both electricians and plumbers to make the proper connections. If the installation crew includes both electricians and plumbers, then "Install HVAC Equipment" may be an appropriate activity. But if the electricians and the plumbers are separate crews, then the activity must be redefined as "Install Electrical for HVAC" and "Install Plumbing for HVAC" so that the work done by each crew may be correctly represented on the RSM diagram.

There are also instances where several activities within a repetitive unit will require the same resource. If this is the case, it may be necessary to group the several activities into one common activity using that resource to avoid the appearance of interruptions in resource usage between units. As an example, consider the steel erection of the warehouse divided into six divisions cited above. In each division, activities performed by a steel erector might consist of erecting columns, erecting trusses, installing bracing, placing roof purlins, and placing wall girts. Even though each of these activities can be considered a separate activity, the same erection crew will be needed for all, and the entire set of activities in one division will be completed before moving on to the next division. Separate activities will be improper and a single activity, "Erect Steel," should be used with the erection crew as its resource.

There are two important and often confused production rates associated with each activity, a *resource production rate* and a *unit production rate*. The *resource production rate* for an activity, rpr_A, is the amount of work that can be accomplished by the resource in one time period. In equation form:

$$rpr_A = \frac{Q_{Ai}}{T_{Ai}} \tag{1}$$

where rpr_A is the resource production rate; Q_{Ai}, is the quantity of work in activity, A, in any repeating unit, i; and T_{Ai} is the time needed to complete the A activity in unit i. Equation 1 is most often used to estimate the activity duration, T_{Ai}, inasmuch as the quantity of work, Q_{Ai}, is taken from the plans and specifications and a standardized resource production rate, rpr_A, for the selected resource and method is taken from company databases or from any of several construction guides in common use in the construction industry.

The *unit production rate* is the number of repetitive units that can be accomplished by a resource during a unit of time. For an activity, A, in any repeating unit, *i*, the *unit production rate*, upr_{Ai}, can be expressed as:

$$upr_{Ai} = \frac{1}{T_{Ai}} \tag{2}$$

where T_{Ai} is the time needed to complete the unit. The unit production rate (and not the resource production rate) is the slope of a production line in an RSM diagram.

If Equation 1 is solved for T_{Ai}, substituted into Equation 2, and applied to any repeating unit, we obtain:

$$upr_{Ai} = \frac{rpr_A}{Q_{Ai}} \tag{3}$$

Observe that the unit production rate is directly proportional to the activity's resource production rate and inversely proportional to the quantity of work in the unit. For example, if rpr_A is expressed in square meters per day (m^2/d) and Q_{Ai} in square meters per floor (m^2/fl), then upr_{Ai} is in floors per day (fl/d). The resource production rate is an attribute of the resource and thus remains constant in any unit involving the same activity (i.e., the same crew will work at the same rate in every repeating unit regardless of the quantity of the work in the unit). Thus, upr_{Ai} may change from unit to unit as a function of the quantity of work, Q_{Ai}, though rpr_A does not.

Sometimes the quantity of work in activities that repeat from unit to unit is not the same in every unit (e.g., “Carpeting for Floor 2” may be twice as much as “Carpeting for Floor 1”). In such instances, the unit production rates will vary depending upon the amount of the work in each unit. For example, let Activities C1, C2, and C3 represent a case wherein the work quantity (e.g., the amount of carpet to be laid on each floor of a multistory project) in Unit 2 is twice that in Unit 1, and the quantity in Unit 3 is one half that of Unit 1. In equation form:

$$Q_{C1} = \tfrac{1}{2} Q_{C2} = 2Q_{C3} \tag{4}$$

The unit production rates are then 1/2 unit per day (1/2 u/d) for activity C1, 1/4 u/d for C2, and 1 u/d for C3, or:

$$upr_{C1} = 2upr_{C2} = \tfrac{1}{2}upr_{C3} \tag{5}$$

This means that the production line for the C activities consists of three linear segments, one for each unit, each having a different slope given by the corresponding upr_{ci}.

BASIC RSM CONCEPTS

Before attempting to construct a complete RSM schedule for a project it is necessary to examine some of the basic concepts that are related to the individual elements of an RSM diagram. Precedence CPM scheduling concepts are used as the foundation upon which RSM scheduling techniques will be developed.

Finish to Start Relationships in RSM with Convergence

Figure 2*a* represents a pair of activities, A1 and B1, removed from a precedence network drawn for a project's Repetitive Unit 1 where the link relationship between the activities is finish to start (FTS). The time duration, *T*, the resource designation, *R*, the Early Start Day, *ESD*, and the Early Finish Day, *EFD*, are as shown in the legend. The values of *R* are expressed as alphabetic symbols to identify the particular resource being used by the activity.

These two activities are plotted as a bar chart in Figure 2*b*. They are plotted again in the form of an RSM diagram in Figure 2*c*. There is only one repetitive unit, and the zero point on the Y-axis is designated by S to indicate the start of the unit. The finish of the unit is designated by F.

The inclined line drawn from the start of Activity A1 in Unit 1 to the finish of Activity A1 in Unit 1 represents the production line for Activity A1. In a similar manner, the production line for Activity B1 is drawn from its start at the end of Day 13 and the start of the unit to its finish at the end of Day 15 and the finish of the unit. The FTS precedence relationship between the activities is indicated by the dotted arrow at Day 13 drawn downward from the finish of Activity A1 to the start of Activity B1. Note that the unit production rate for Activity A1 is 1/3 unit per day (1/3 u/d), and for Activity B1 is 1/2 u/d. These rates will be recognized as the mathematical slopes of the respective production lines.

The same pair of activities extended over three repetitive units is plotted in the form of a bar chart in Figure 3*a*. Each unit contains the two activities, and the numeral associated with each activity identifies the unit in which it is scheduled. Since only technical precedence logic is employed, there is a one day lag between the B activities from one unit to the next.

Figure 3*b* shows a unit-by-unit RSM plot of the same activities with the FTS relationships shown by the downward pointing arrows. The production lines plotted for Activities A1 through A3 form a continuous straight line beginning at the end of Day 10 and ending on Day 19. Because each A activity uses Resource K and has a unit production rate of 1/3 u/d, it follows that the production line for the three A activities also has the same unit production rate of 1/3 u/d. While no attempt was made to provide for the continuous utilization of the resource from unit to unit, the continuous production line for the A activities ensures that this is true.

The production lines for Activities B1 through B3 do not form a continuous production line when plotted unit by unit because of the lags between the B activities. To make a continuous production line for the B activities and provide for the uninterrupted utilization of resources, the start of Activity B1 must be delayed by two days and the start of Activity B2 must be delayed by one day. The resulting production line for the B activities is shown in Figure 3*b* as a dashed line beginning at the end of Day 15 and continuing through Unit 2 then extending as a solid line through Unit 3 to finish on Day 21. Because the unit production rate for each unit's B activity is 1/2 u/d, the unit production rate of the production line for the B activities is also 1/2 u/d.

Notice that the two continuous production lines converge toward the finish of Unit 3 because the unit production rate of the B line is greater than that of the A line. Also note that at Day 19 and the beginning of Unit 3, the end of the FTS arrow between the finish of Activity A3 and the start of Activity B3 controls the start of Activity B3, and subsequently, the position of the B line. This location, or control point, has been labeled $cp_F(AB)$ where the subscript, F, stands for finish and signifies the last unit in the sequence, and the letters A and B show the dependency of Activity B upon Activity A. This illustrates a basic RSM principle:

> ***When the unit production rate of an activity's production line is greater than the unit production rate of the preceding activity's production line, the two production lines will tend to converge as the number of units increases. Owing to the desired continuous utilization of resources from unit to unit, this convergence tends to place any dependency control between the activities toward the last unit in the sequence.***

With the above principle in mind, a simple procedure for constructing the production line for the B activities suggests itself. First establish the control point $cp_F(AB)$ at the start of Activity B3 and then draw the continuous production line for B through it.

Since Activity B3 is the last activity in the sequence, another control point, called cp_E, at the end of Activity B3 can also serve as a point through which the B production line may be drawn. (The subscript E in cp_E stands for the end of the activity and the production line.)

The two days shown in Figure 3*b* between the end of Activity A3 and the end of Activity B3 at cp_E is a lead time, LT, that relates the finish of the B production line to the finish of the A production line. This corresponds to a finish to finish (FTF) relationship shown in the equivalent CPM overlapping diagram of Figure 3*c* where the lead time for the link between the A and B activities represents the amount of time that must remain in the B activity after the finish of the A activity. In this context, the two day duration of Activity B3 represents the amount of time that must remain in B after the finish of Activity A3 and sets the lead time at 2 days. Thus, the control point, cp_E, can be positioned two days after the finish of Activity A3, and the B production line can be drawn through it.

Finish to Start Relationships in RSM with Divergence

Figure 4*a* is similar to Figure 3*a*. It is a bar chart of another pair of activities removed from a precedence network for a project's Repetitive Unit 1. These two activities are extended over three repetitive units with the activities grouped by unit. Each A activity has a duration of 2 days and each B activity has a duration of 3 days. The precedence relationship between the activities in each unit is FTS, and each activity is shown in its scheduled early start position when only technical precedence logic is used. Thus, there is a lag of one day between Activities A2 and B2, and a lag of two days between Activities A3 and B3.

Figure 4*b* is an RSM unit-by-unit plot of the same activities, with the FTS relationships indicated by the downward pointing dotted arrows at Days 12, 14, and 16. The lags shown between the finish of each A activity and the start of its related B activity are the same as those shown in Figure 4*a*. As plotted in Figure 4*b*, the production lines for both A and B are continuous and ensure the uninterrupted utilization of resources even though no deliberate attempt was made to achieve resource continuity.

Also note that in Figure 4*b*, the unit production rate of the B production line, 1/3 u/d, is smaller than the unit production rate of the A production line, 1/2 u/d. The two unit production lines therefore diverge and the FTS control between the two is found at Day 12 in Unit 1. This control point is labeled $cp_S(AB)$, where the subscript, S, stands for start and signifies the first unit in the sequence, and the letters A and B show the dependency of Activity B upon Activity A. This illustrates another basic RSM principle:

> ***When the production rate of an activity's production line is smaller than the production rate of the preceding activity's production line, the two production lines will tend to diverge as the number of units increases. Owing to the desired continuous utilization of resources from unit to unit, this divergence tends to place any dependency control between the activities toward the first unit in the sequence.***

The two days shown in the figure between the start of Activity A1 and the start of Activity B1 at $cp_S(AB)$ is a lead time, LT, that relates the start of the B production line to the start of the A production line. This corresponds to a start-to-start (STS) relationship shown in the equivalent CPM overlapping diagram of Figure 4*c* where the lead time for the link between the A and B activities represents the time to accomplish the work required in the A activity before the B activity can begin. In this context, the two day duration of Activity A1 represents the amount of time that must elapse before the start of Activity B1 and sets the lead time at 2 days. Thus, $cp_S(AB)$, can be positioned two days after the start of Activity A1. The B line passes through $cp_S(AB)$, ends at the finish of Activity B3 on Day 21, and sets the duration of the project at 21 days.

Effects from Changing Unit Production Rates on FTS Activities

Suppose that the crew for each B activity of Figure 4 is increased by fifty percent. This change reduces each B activity duration to two days and increases each unit production rate to 1/2 u/d, the same as that of each A activity. An RSM diagram for the three units of Figure 4*b* with this revised unit production rate is shown in Figure 5*a* along with the dashed production line from Figure 4*b*.

The control point, $cp_S(AB)$, still controls the position of the B production line which now lies parallel to the line for the A activities. Therefore, increasing the unit production rate of the B production line from 1/3 u/d to 1/2 u/d is tantamount to rotating the production line about this control point. A curved arrow at $cp_S(AB)$ signifies this rotation. The project duration is reduced from 21 days to 18 days, and the FTS arrow at the beginning of Activity B3 defines another control point, $cp_F(AB)$, through which the new B production line passes.

If the resources of each B activity are doubled over those shown in Figure 5*a*, the unit production rate of the B activities becomes 1 u/d and causes the A and B lines to converge. A further rotation of the B production line about $cp_S(AB)$ would violate the FTS relationships at Days 14 and 16, so the control of the B line must shift to $cp_F(AB)$ at the beginning of Unit 3. Figure 5*b* shows this shift in control point and the rotation of the B production line about $cp_F(AB)$ where the curved arrow refers to the rotation of the line. The B line now begins at the end of Day 14 and sets the project duration at 17 days, one day shorter than in Figure 5*a*.

Start to Start Relationships in RSM with Convergence

Two activities, A1 and B1, removed from a precedence network for Unit 1 of a repetitive project are depicted in Figure 6*a*. The start to start (STS) link relationship with its lead time of two days implies that the B1 activity cannot start until two days after the start of the A1 activity. The two activities are plotted as a bar chart in Figure 6*b*, where the lead time is indicated by the hatched portion of Activity A1. A plot of the production lines for the same activities is shown in Figure 6*c*, and the lead time of two days is shown between the finish of Day 10 (the start of Activity A1) and the finish of Day 12 (the start of Activity B1). The unit production rates for these lines are 1/6 u/d for Activity A1 and 1/3 u/d for Activity B1.

The lead time shown is between the start of Activity A1 and the start of Activity B1 and relates only to time. An alternative for establishing the start of Activity B1 can be based on the work production of Activity A1. For example, the start of Activity B1 can only begin after one third of the work on Activity A1 has been completed and an apparent FTS relationship exists at this point. A downward pointing arrow has been inserted at Day 12 to emphasize this relationship.

It is assumed that once an activity has started it will be performed continuously until it finishes. Activity A1, for instance, is expected to continue over Days 11 through 16, and Activity B1 is expected to continue over Days 13 through 15. Hence, the crossing of these production lines has no significance.

Figure 7*a* shows a unit-by-unit RSM plot of the same pair of activities extended over three repetitive units. The production lines for the A activities are plotted as solid lines and the production lines for the B activities are plotted as dashed lines. As a result of this plot, the production lines for the A activities are continuous from the end of Day 10 to the end of Day 28, and this line represents the production line for the A activities through all three units. This line has a unit production rate of 1/6 u/d.

The production lines for the unit-to-unit B activities are interrupted and there is a FTS lag between Activities B1 and B2 and also between Activities B2 and B3. Because the unit production rate for the B activities, 1/3 u/d, is greater than the unit production rate for the A activities, there is convergence and the control point between the lines A and B is found at the largest numbered unit, Unit 3. The control point, $cp_F(AB)$, is located by adding the lead time of two days to the scheduled start time for Activity A3. The production line for the B activities with continuity of resources passes through the control point and extends from the finish of Day 18 through Day 27.

This production line is the *earliest* line that satisfies technical precedence, resource availability, and resource continuity constraints, but it is not the only line that meets these requirements. In Figure 7*a*, there is a one-day total float, TF, for the B production line shown between the finish of Activities B3 and A3, and any B production line that finishes within this float will also satisfy all the constraints. Figure 7*b* shows the A and B production lines with the line for the B activities shifted to finish at the end of Day 28. This new scheduled production line for the B activities is the *latest* line that meets the constraints. Notice that each A activity now has a STS lag which was created when the B activities were positioned to satisfy the continuity of resources.

Start to Start Relationships in RSM with Divergence

Another pair of activities removed from a precedence network for Unit 1 is depicted in Figure 8*a*. The link relationship between them is start to start (STS) with a lead time of two days. The two activities are shown in bar chart form in Figure 8*b* where the lead time is indicated by the hatched portion of Activity A1. The production lines for the same activities are shown in Figure 8*c* where the lead time is shown between the finish of Day 10 and the finish of Day 12. The unit production rates for these activities are 1/6 u/d for Activity A1 and 1/8 u/d for Activity B1, so the two production lines diverge.

The RSM diagram for these same activities repeated over three units is shown in Figure 9. Because the production lines diverge, the control point between the lines is found at Unit 1 which is the smallest numbered unit. The control point, $cp_S(AB)$, is located by adding the lead time of two days to the scheduled start time for Activity A1 or its equivalent FTS relationship from one third of the work on Activity A1. This is shown by the dotted arrow at Day 12 between the end of the first two days of Activity A1 and the start of Activity B1. The production line for the B activities passes through $cp_S(AB)$ and extends from the finish of Day 12 through Day 36. Because the production lines diverge, STS lags exist between activities A2 and B2 and between activities A3 and B3.

Increasing Unit Production Rates on STS Activities

Assume that the 36 day duration of the three units is unacceptable. The B activities clearly control this duration and their resource production rates will need to be increased if the duration is to be reduced. Suppose further that the crew size for each B activity is increased such that the unit production rate of the B production line is equal to the rate for the A production line or 1/6 u/d.

The RSM diagram for the three units with this revised unit production rate is shown in Figure 10*a* along with the B production line from Figure 9. The control point, $cp_S(AB)$, at the start of the first unit and the end of Day 12 still controls the position of the production line for the B activities, and the line becomes parallel to the line for the A activities. This new line ends at Day 30 and sets a duration for performing the three units at 30 days, six days less than the 36 days originally planned.

Increasing the unit production rate of the B production line is tantamount to rotating the production line about the control point, $cp_S(AB)$, where a curved arrow is shown to emphasize this rotation. Note that the virtual FTS relationship is maintained for all the A activities as indicated by the downward arrows at Days 12, 18, and 24. In Unit 3, the virtual FTS relationship and the lead time set another control point, $cp_F(AB)$, that could be used to draw the production line for the B activities.

Now assume that further time reduction is desired for the finish of these three units, and that the resource production rate of the B activities is increased until the unit production rate is 1/3 u/d. A further rotation of the B production line about the $cp_S(AB)$ would violate the STS relationship for Units 2 and 3, so the control between the production lines shifts to $cp_F(AB)$ at Day 24 and the start of Unit 3. Figure 10*b* depicts this new situation, and it can be seen that the production line for B now begins at the end of Day 18 and ends on Day 27. The curved arrow at $cp_F(AB)$ refers to the rotation of the line. The similarity between this figure and Figure 7*a* should be noted.

Also shown in Figure 10*b* are the positions of the B activities and the lags between them if resource continuity had not been recognized. Because the technical precedence and resource availability constraints are satisfied, a planner could choose to begin Activity B1 at any time between the end of Day 12 and the end of Day 18, but there would be a time interruption of the resource between Activities B1 and B2 or between Activities B2 and B3. Therefore, the start of the B production line at the end of Day 18 must be considered as a schedule time that is assigned to satisfy the resource continuity constraint.

Finish to Finish Relationships in RSM with Convergence

Two activities, A1 and B1, that have been removed from a precedence network for one unit of a repetitive project are depicted in Figure 11*a*. The link relationship between them is finish to finish (FTF) with a lead time of two days. This signifies that after Activity A1 has been completed, Activity B1 still has two days of work remaining to be completed. These same activities are shown in bar chart form in Figure 11*b* where the lead time is indicated by the hatching on Activity B1. The same two activities are shown as an RSM diagram in Figure 11*c* where the lead time is shown between their finish days.

The lead time shown between the finish of Activity A1 and the finish of Activity B1 relates only to time. An alternative for establishing the relationship between the activities can be based on the amount of work in Activity B1. Figure 11*b* signifies that after Activity A1 has been completed, Activity B1 still has two days of work remaining to be done. In a sense, this part of Activity B1 can be considered a separate activity and in Figure 11*c*, a virtual FTS relationship can be inserted between the finish of Activity A1 and the start of the last two days of Activity B1. A downward pointing arrow has been drawn at Day 18 to show this relationship.

An RSM diagram showing the continuous production lines for these two activities extended over three units is illustrated in Figure 12. The unit production rates are 1/8 u/d for the A production line and 1/6 u/d for the B line. Because the unit production rate for the B activities is greater than that of the A activities, the production lines converge and the control between them is found at Day 36 in Unit 3. This control point, $cp_{FT}(AB)$, has been located by adding the lead time of two days to the scheduled finish time for Activity A3. (The FT subscript to the control point refers to the finish of the production line and that the control point was determined by using the lead time.) Another control point, $cp_{FW}(AB)$, may also be found by inserting a virtual FTS relationship at Day 34 in recognition of the two days of work that must remain in Activity B3 after Activity A3 is finished. (The FW subscript to the control point refers to the start of the production line and that the control point was found using the work content of the activity.) The continuous production line for the B activities passes through both control points. Note that there are FTF lags between the A and B activities at the finish of Units 1 and 2, and that these lags precede their lead times.

Finish to Finish Relationships in RSM with Divergence

Two other activities removed from a CPM network for Unit 1 are depicted in Figure 13*a*. The link relationship is finish to finish (FTF) and the lead time is two days. Notice that the *ESD* of Activity B1 is eight days and that for Activity A1 is ten days. These starting dates are in agreement with the usual precedence network calculations.

The two activities are shown in bar chart form in Figure 13*b* and the lead time has been hatched as before. Figure 13*c* shows the two activities drawn in RSM form. The intersection of the production lines for these activities has no particular significance since it results from the differing production rates and starting times.

An RSM diagram for three units has been constructed in Figure 14. The unit production rate for the A activities is 1/4 u/d and for the B activities is 1/8 u/d. Because the unit production rate for the B activities is less than that of the A activities, the production lines diverge and the control point is found in Unit 1. Using the amount of work concept discussed in connection with Figure 11*c*, the control point is located by considering a virtual FTS relationship between the finish of Activity A1 and the start of the last two days of Activity B1. This control point is marked cp_{SW}(AB) at Day 14. An alternative and equivalent approach to locating the control point would be to use the lead time and establish another control point, cp_{ST}(AB), at Day 16.

Figure 14 also has the lag between the finish of Activity A3 and the finish of Activity B3 defined. This eight day lag is equivalent to the total float of Activity A3 if the three units were a complete project and the customary CPM calculations were made. In a similar manner, the total float of Activity A2 can be determined as four days even though it is not marked on the diagram.

Given that Activities A2 and A3 have floats, it might be assumed that the production line for the A activities could be shifted to a later time. This is not so because resource continuity requires that the technical restraint at Day 14 must be kept. Activities A2 or A3 could be scheduled at later times if desirable for other management purposes, but their resources would be interrupted. The production line shown is therefore the latest line that meets all three constraints.

Increasing Unit Production Rates on FTF Activities

Assume that the 32 day duration of the three units planned above is not acceptable. The B activities determine the critical path and their resource production rates will need to be increased if the duration of the three units is shortened. Now suppose that the crew for each B activity is increased with the result that the durations of the B activities are reduced to four days and the unit production rates are increased to 1/4 u/d, which are the same as those of the A activities. The RSM diagram for three units with these revised unit production rates is shown in Figure 15*a* along with the production line from Figure 14.

In Figure 14 both control points cp_{SW}(AB) and cp_{ST}(AB) could be used to construct the B production line because one quarter of the work remaining in a B activity is equal to a two-day lead time. Because the duration of each B activity is now only four days, it will take one half the work remaining in a B activity to equal a two-day lead time.

Either control points cp_{SW}(AB) or cp_{ST}(AB) could be used for rotation of the B line, but a decision must be made as to whether the time concept or the work concept is to be retained. For example, if the original work concept is retained, then the cp_{SW}(AB) control point would be the proper one to use, but if the original lead time concept is retained, then the cp_{ST}(AB) control point is the appropriate one. In Figure 15, the time concept must be chosen because a rotation about cp_{SW}(AB) would cause a violation of the FTF lead times.

The B production line has been rotated about $cp_{ST}(AB)$ and a curved arrow signifies this rotation. The line now begins at the end of Day 12 and finishes on Day 24. Note that the lead time relationship is maintained for all the A activities. In Unit 3, the lead time also sets another control point, $cp_{FT}(AB)$, that could be used to draw the production line for the B activities.

Now assume that the unit production rate of the B activities is increased further to 1/3 u/d. Rotation will take place about control point $cp_{FT}(AB)$ at Day 24 as shown in Figure 15*b*, and the B production line will extend from the end of Day 15 to Day 24. No reduction in the duration of the three units has been made, but lead times have been created for the A activities.

DIAGRAMMING CONVENTIONS

In the above discussions only three repetitive units were used so that the relationships could be clearly seen. Such a small number of units probably would not be found in more realistic projects, and when there are a large number of units in the project, some conventional representations may be adopted.

To illustrate a few of these conventions, consider the RSM diagram shown in Figure 16. The figure depicts the production lines for two chains of activities for a twenty-story building. The Y-axis numbers are the building floors. No attempt is made to make explicit the start and finish of each floor, but it may be assumed that the numbers refer to the finish of each floor. The X-axis shows the ends of working days as has been shown in the previous diagrams.

It has been assumed that the precedence diagram for one unit is the same as the FTS diagram of Figure 2*a*. The repetitive units are the building floors, of course, and the unit production rates are 1/3 fl/d for the A activities and 1/2 fl/d for the B activities. The two production lines are converging because of the relationship between the production rates.

The production line for the A activities begins at zero and ends on Floor 20 and Day 60. When all the A activities are finished, the last B activity must still be completed, and control point, $cp_{FW}(AB)$, exists at the end of Day 60 and the start of Floor 20. This is based on the work to be completed concept. Its equivalent control point, $cp_{FT}(AB)$, based on the lead time concept, can also be identified, and in this case, a FTF lead time equal to the duration of Activity B20 is added to the finish of the A line at Floor 20 to mark the end of the B activities. The recognition of this equivalence is common, and the production line for the B activities is constructed from this point at Day 62. The B production line therefore extends downward from Day 62 to Day 22.

Also shown in Figure 16 are the early positions of the B activities for the first five floors if the activities on each floor were completed separately, but these positions do not produce continuity of the B resources. Also the B production line is critical because any delay at any floor would delay the project. The time differences between the early B positions and the B production line represent floats that could be used by the constructor for management purposes associated with cost or resource control.

DIAGRAMS WITH LENGTH UNITS

In the previous paragraphs the assumption was made that the repetitive units were discrete. Each one contained one or more activities, had defined start and finish points, and an activity in one unit was completed before the same activity in the next unit was begun. These assumptions are not as clearly articulated when length units are used to model projects such as highways, railroads or pipelines, yet the same principles are employed. In the types of projects mentioned, repetitive units are generally small and numerous. Usually each unit will have the same set of activities, and one activity of the set will be performed continuously over several units before the next activity is begun.

Figure 17 represents the logic that must prevail in every unit of a highway project. For simplicity, only three activities, A, B, and C, are shown along with their respective resource production rates. Because only logic is depicted, there are no activity durations or early start and finish times indicated.

On the link between Activities A and B, a lead time of two days is shown. This is the minimum time that must lapse at every unit between the finish of Activity A and the start of Activity B. For example, such a lead time might represent the time needed for the curing of Activity A’s material. This would be a time restraint on Activity B because Activity B could not start until this curing has taken place. This lead time is frequently called a *time buffer*.

On the link between Activity B and Activity C there is shown a lead distance, LD, of 40 stations. (A station is 100 feet long and is commonly used as a measure of length along a highway.) For example, it may be desirable, or even necessary, to limit the distance between two pieces of equipment moving in the same direction. Often these pieces will be performing different activities and so physical space is needed between them. This distance is often called a *distance buffer*.

A portion of the highway project’s RSM diagram is depicted in Figure 18. It contains only the three activities of Figure 17. Because length units are being used, the X-axis is designated as stations and the Y-axis is expressed in working days. The project is 20,000 feet in length, or 200 stations, and the progression of activities is now upward along the time line.

The production line for the A activities is constructed beginning at time zero and ending at Station 200+00 and Day 10. Its unit production rate is 20 stations per day. The continuous nature of the activities leads to a common convention to call the production line for all the activities A as the production line for Activity A. This convention will be followed in the remainder of this discussion.

The production rate for Activity B is calculated as 40 stations per day. This rate is greater than that of Activity A so the production line converges toward the end of the project at Station 200+00. Although the unit axis is marked off in stations, any small distance along this axis may be called a unit. For example, if one inch is considered as a unit, it would not be distinguishable at the scale of the diagram. Following the principles previously discussed, the control point would be located at one unit, or one inch, less than Station 200+00 at Day 10. At the scale being used, it would be essentially at Station 200+00. The production line for Activity B is drawn through this control point, $cp_F(AB)$, so Activity B finishes on Day 12 and starts at Day 7.

The unit production rate for Activity C is also 40 stations per day. This is the same as that of Activity B and the two production lines will be parallel. The lead distance between the two activities has been defined as 40 stations which implies that Activity C may start as soon as Activity B has completed 40 stations. This lead distance, or buffer, has been shown on the figure as a dotted line between Stations 0+00 and 40+00, and locates a control point, $cp_S(BC)$, at Day 8 and Station 0+00. When Activity C has been completed at Station 200+00, the finish of all three activities is found to be 13 days.

Unit Production Rate Adjustments in Diagrams With Length Units

In planning projects that use length units it often happens that unit production rates need to be adjusted to accomplish some planning objective. The procedure for making these adjustments is the same as has been illustrated for projects with discrete activities. Two cases, one with an increased rate and another with a decreased rate, are shown in Figure 18 to illustrate the process.

For the first case, assume that the unit production rate for Activity C is increased to 80 stations per day. Because of the difference in production rates, the control point for rotation of the production line will tend to be toward the project’s finish. The lead distance buffer between Activities B and C must be maintained, and the control point is found by moving back 40 stations from the finish of Activity B. This point has been labeled $cp_F(BC)$, and the adjusted production line through it has been labeled C’. If this C’ line is used, the duration of the three activities would decrease to 12.5 days.

In the second case, assume that the unit production rate for Activity C is decreased to 20 stations per day. The control point for rotation is then toward the beginning of the project and is found at Station 0+00 and Day 8; this point is the same as used when the unit production rate was 40 stations per day. The new line, C", has been drawn through this point. Note that the lead distance buffer between Activities B and C" is always satisfied. Using Activity C" increases the duration of the three activities to 18 days.

RSM DIAGRAM CONSTRUCTION

The above examples use only a few production lines so that basic RSM principles are clearly illustrated. However, their small size is not sufficient to demonstrate the construction of an RSM diagram, and a project with six repeating units, each having six discrete activities, has been chosen for this purpose.

The CPM precedence diagram for the activities in the first unit is shown in Figure 19, where all the link relationships are finish to start. The solution for the early start and finish days, the critical path, and the 12 day duration is shown on the network. Of this information, only activity durations and precedence relationships are required for the construction of an RSM diagram. Similar diagrams, not shown here, establish these same requirements for each of the remaining units.

Figure 20 is the RSM diagram for all the activities in the project. Since the first activity in the precedence diagram is A, the first production line plotted in Figure 20 is that for the A activities. The unit production rate for the A production line in Units 1 and 2 is 1/2 u/d. However, the amount of work to be done in Units 3 and 4 by the A activities is twice the work to be done in Unit 1. Hence, under the assumption of constant resource production rates, the unit production rate of the A production line in Units 3 and 4 will be 1/4 u/d. The amount of work in Units 5 and 6 is the same as in Unit 1 and the unit production rate for the A line is again 1/2 u/d. The A production line that begins at time zero and ends at Day 16 consists of three connected linear segments with different slopes. Even so, the continuity of these segments ensures the continuous utilization of the resource needed by the A activities.

In the precedence diagram of Figure 19, the B and C activities are not related, but each is a successor to the A activity. The next choice for plotting a production line is therefore arbitrary, and the line for the B activities is selected. The FTS relationships between the A and B activities must prevail at every unit, although there may be link lags between these activities at any unit.

The unit production rate for each of the B activities is 1 u/d which is greater than either of the unit production rates for the A activities, so the production lines converge. The B production line is therefore controlled at the start of Unit 6 where the control point, $cp_2(AB)$, is shown at the dotted FTS arrow on Day 16. To maintain the continuity of resources for the B activities, the production line for the B activities must pass through this point.

It sometimes happens that an interruption in resource continuity may need to be planned to meet some known or predicted circumstance. In this instance, the B activities are performed by a subcontractor from a different area, and on each trip to the site, the subcontractor’s truck can deliver materials sufficient for completing only three units. The production line for the B activities is therefore interrupted between Unit 3 and Unit 4 to accommodate the delivery, and the B line has two segments.

Control point $cp_2(AB)$ can continue to control the position of the upper segment of the B production line, but another control point, $cp_1(AB)$, controls the leftmost possible position of the lower segment. This point is located at the FTS arrow on Day 8 in Unit 3, and the production line for the B activities in the first three units has been drawn through it to maximize the break time available to the subcontractor. The production line for the first three B activities is now planned to start at the end of Day 6. The dotted line labeled "Work Break" between Days 9 and 14 illustrates the planned interruption.

The production line for the C activities is plotted next. The unit production rate for all C activities is 1/4 u/d which is smaller than, or equal to, that of the A activities. Hence, the production lines for the A and C activities diverge and the plot of the production line for the C activities is controlled at Unit 1. The FTS relationship between Activity A1 and Activity C1 is shown by the downward dotted arrow at Day 2. There is a lead time shown on the link between the A and C activities in Figure 19, and this lead time is shown between Days 2 and 4 in Figure 20. Lead times must prevail in each repeating unit, but since the production lines diverge, only the lead time in the first unit needs to be recognized here. Consequently, the control point, cp(AC), is found at the end of Day 4 in Unit 1 and marks the beginning of the production line for the C activities.

This project does not have an Activity C in Unit 5, so when Activity C in Unit 4 is completed, Activity C in Unit 6 can begin, and the C production line shifts upward as indicated by the dotted line in Unit 5 at Day 20. This displacement interrupts the C production line at Unit 5, but does not create an interruption in the continuous utilization of its resource. It may be observed that the production lines for the B and C activities appear to cross each other. This has no significance because the two activities are not related to each other.

The next production line chosen for plotting is for the D activities. It is selected because Activities B and C are the predecessors to Activity D in Figure 19, and because these two production lines have been drawn already and at least one of them will control the position of the D line.

All possible control points from each predecessor production line must be considered when establishing a control point to position a production line. First, consider the relationships between the B and D production lines. The unit production rate for the B line is 1 u/d and for the D line is 1/3 u/d, so these lines diverge. One possible control point, labeled cp_1(BD), might be located at Day 7 and the start of Unit 1. Because of the work break, another possible location for a control point might be at cp_2(BD) at Day 15 and Unit 4. Now consider the relationship between the C and D production lines. The unit production rate for the C line is 1/4 u/d and for D line is 1/3 u/d, so the production lines converge. Because of the skip in units on Day 20 and the discontinuity in the C line, there are two possible control points; one located at Day 20 and the start of Unit 4, and the other at Day 24 and the start of Unit 6.

If the control point at Day 7 is used to control the position of the D line, none of the FTS relationships between Activities B and D would be violated, but all the relationships between Activities C and D would be. Further, if the control point at Day 15 is used, the FTS relationship between Activities B and D would be violated in Unit 1 in addition to all those between Activities C and D. If the control point at Day 24 is used, the FTS relationships between all the B and D activities can be met, but the relationships between C and D cannot be met at Units 3 and 4. The remaining possibility is to use control point cp_1(CD) at Day 20 and the start of Unit 4. This point allows all FTS relationships to be satisfied, and the production line for the D activities is drawn through it.

The production line for the E activities is considered next. Its unit production rate is 1 u/d, and it is related only to the production line for the C activities. As before, two control points are possible, one at Unit 4 and the other at Unit 6. In this instance, control point cp(CE) is found in Unit 6 at Day 24, and the E production line is drawn through it. The fact that the production lines for the E and D activities intersect is of no consequence since they are not related to each other.

The production line for the F activities is the last to be drawn. It depends upon the lines for activities D and E. The unit production rate for the F line is 1 u/d which is larger than, or equal to, either of the rates for these predecessors. This convergence forces the control point to be in Unit 6. Because the production line for the D activities finishes later than that of the E activities in Unit 6, control point cp(DF) is clearly established at Day 29 in Unit 6. The finish of the production line for the F activities marks the end of the project and sets the project duration at 30 days.

THE CONTROLLING SEQUENCE

In CPM networks, a critical activity is defined as one that, if delayed, will delay the project, and a chain of these critical activities extending from project start to project finish is called the critical path. Adding the durations of the critical activities along this path establishes the minimum project duration consistent with the technical precedence and resource availability constraints explicitly expressed in the network. However, the determination of the project duration from a critical path does not apply in RSM because of the additional resource continuity requirement. This requirement forces noncritical activities to become critical, and may cause noncritical activities to be included in the chain of activities that controls project duration.

In RSM, the chain, or sequence of activities, that establishes the minimum project duration is called the *controlling sequence*. This sequence maintains all technical precedence, resource availability, and resource continuity constraints, and passes through control points which switch the sequence from production line to production line. Some of the activities on the controlling sequence may be critical in the CPM sense, and some may not. If the activity is critical, a delay in the completion of the activity delays the completion of the project. If the activity is noncritical, a delay in the completion of the activity does not delay the completion of the project, but introduces discontinuities in resource utilization.

The controlling sequence through the six unit project of Figure 20 is found by tracing along production lines from the project finish to the project start while shifting from one production line to the next at the defined control points. The trace begins at the finish of the F production line at Day 30. It moves downward along the F line to control point cp(DF). The trace then shifts to the production line for the D activities in Unit 6 and moves downward again to control point cp_1(CD). It shifts again to the C line and moves down to control point cp(AC) at Day 4. At this point it moves through the lead time and the FTS arrow to the A production line in Unit 1. The trace finishes at Day zero which is the start of the A line.

The individual activities that comprise this controlling sequence are Activity A in Unit 1; Activities C in Units 1, 2, 3, and 4; Activities D in Units 4, 5, and 6; and Activity F in Unit 6. These activities also happen to be critical (i.e., none can be delayed without delaying the project), and they form a critical path that is the same, in this instance, as the controlling sequence. These activities are shown in Figure 20 by a heavy solid line.

Other activities are critical only because they are scheduled to provide resource continuity. For example, Activities D1, D2, and D3 are each scheduled to start at Days 11, 14, and 17, respectively, to ensure the continuous usage of their resource. If the completion of any one of them is delayed, then part of the controlling sequence and the completion of the project will be delayed. Activities D1, D2, and D3 are therefore critical by definition only because of their RSM schedule. Similarly, Activities F1 through F5 are critical because they are scheduled to provide continuous usage of their resource. Activities such as these are called *resource critical activities* and they are shown in the figure with heavy dashed lines.

Activities A2 through A6, B1 through B6, C6, and E1 through E6 are not critical. A delay in any one of these will not delay the project, but may cause an interruption in the continuity of resources from unit to unit.

In projects that contain production lines derived from start-to-start or finish-to-finish activities, two types of control points were depicted. One type was based on time alone as signified by the lead time, and the other was based on the work content in the lead-time segment of one of the activities. In the determination of the controlling sequence, the control point associated with the work content must be used as the point to switch from one production line to the next, because only the work in the lead-time segment can belong to the controlling sequence or to the critical path. The reader should refer to Figures 9 and 12 where these conditions are illustrated.

REDUCING THE PROJECT DURATION

The creation of a project schedule deemed satisfactory for construction is an iterative process. The first plan probably is not satisfactory for any number of reasons, and several adjustments to logic, resource usage, resource quantities, and so forth, will need to be made. One principal reason to adjust the plan is to reduce the project duration.

Minimum project durations can theoretically be achieved by adding resources to some activities and subtracting resources from others until all unit production lines have the same unit production rate and are parallel to each other. This ideal minimum, however, cannot always be achieved because construction resources can only be expressed in integer form. For example, there cannot be 3.7 workers on a crew; there can be either 3 or 4. Similarly, there cannot be 1.5 pavers on a highway project; there can be either 1 or 2. Moreover, a production line may be composed of several linear segments that have different slopes because of different quantities of work in each of the repetitive units. Consequently, unit production lines cannot always be parallel, and a more pragmatic approach needs to be employed.

The procedure for reducing the project duration is best illustrated by example. Assume that the duration of 30 days for the project in Figure 20 is not acceptable, and that a review of the schedule is needed to seek a reduction in time. An examination of the figure shows that the C and D activities are major contributors to the 30 day project duration, so it appears that the duration of the project can be reduced if the unit production rate of one of these two production lines is increased.

Suppose further that it is possible to increase the unit production rate of the D production line to 1/2 u/d as shown in the RSM diagram of Figure 21. The A, B, and C production lines are unchanged. The new rate for the D line is greater than the rate for the C line, so the two lines converge as before, and the previously used control point, cp_1(CD), at Day 20, is still the control point for the D line. The line is rotated about this point and the new production line for the D activities causes the D line to pass through both control points, cp_1(CD) and cp_2(CD). Any further increase in unit production rate for the D line will cause a rotation about cp_2(CD).

The E production line remains in its position because its control point, cp(CE), is unchanged. However, the F production line is affected by the rotation of the D line. The new unit production rate of the D line is still less than the rate of the F line, so there is still convergence between the two lines, but the cp(DF) control point has been shifted back to Day 26. The F production line is drawn through this control point and sets the project duration at 27 days, or 3 days less than the original plan.

The controlling sequence in Figure 21 is found by tracing backward from the end of the project, as was done earlier. The trace begins at Day 27 with the finish of the F production line and moves downward to cp(DF) where it shifts to the D line and moves downward until cp_2 (CD) is reached. The trace splits at this point and one branch shifts to the C line while the other branch continues down the D line to cp_1(CD) where it shifts to the C line. The trace for both branches continues down the C line to cp(AC) and then shifts to the A line. Note that this creates two controlling sequences, one passing through Activity C6 and the other through Activities D4 and D5. All the activities on these controlling sequences are critical and also form two critical paths. As before, these controlling sequences and critical paths are shown in the figure by a heavy solid line. Activities D1 through D3 and F1 through F5 are also critical but do not belong to either the controlling sequence or to the critical path, so they are shown by a heavy dashed line.

A PARADOX

As another alternative, suppose the unit production rate of the C line is increased instead of that of the D line because the unit production rate of the C line is the smallest rate of the two. Figure 22 shows the RSM diagram with the rate of the C line increased from 1/4 u/d to 1/2 u/d. This new rate is greater than, or equal to, either of the rates of the A production line, and the lines will converge. This change from divergence to convergence causes a shift of the control point relating the two activities. The A and B production lines remain the same as before, but the new control point, cp(AC), shown in Figure 22, is located at Day 18 in Unit 6.

The unit production rates for the D, E, and F lines also remain as in Figure 20, but because of the change in the rate for the C line, the C and D production lines diverge and the control point, cp(CD), relating lines C and D shifts to Day 12 in Unit 1. The production line for the D activities now begins at the end of Day 12 and ends on Day 30, which causes the project duration to be increased by one day, to 31 days. Thus, increasing the unit production rate of the C activities does not shorten the project duration as expected, but actually increases it. This paradox is explained below.

As before, the controlling sequence of activities in Figure 22 is found by tracing from the end of the project back down along the F production line until control point, cp(DF), at Day 30 is reached. The trace then shifts to the D line and follows downward to control point, cp(CD), at Day 12. At this point, the trace moves up the FTS arrow to the C line which is controlled by cp(AC) at Day 18 and Unit 6. It is important to note that control point, cp(AC), occurs later than cp(CD). As a result, the trace of the controlling sequence traverses the C line forward in time rather than backward. The trace next follows the lead time between Activities A and C and upward on the FTS arrow to the A line. The trace follows down the A line to the start of the project. The controlling sequence consists of the activities represented by the double and heavy continuous lines.

All the A activities and the C activities from Units 2 through 4 are shown with double lines in Figure 22. This notation indicates that these activities are part of the controlling sequence, but are not critical because they have floats that can be utilized if any are delayed. All of the D activities and the F activity in Unit 6 are shown with heavy solid lines as before to illustrate that they are part of the controlling sequence and are also critical. Activity C in Unit 1 and Activities F in Units 1 through 5 are shown with heavy dashed lines to indicate that they are critical, but do not belong to the controlling sequence. The remaining activities in the project are shown with plain solid lines since they are neither critical nor lie on the controlling sequence.

The paradox wherein expediting an activity results in an increase in project duration can be explained by reference to Figure 22. When the unit production rate of the C line was increased, the line rotated counterclockwise and the new upr_C became equal to or greater than upr_A. Hence, control point, cp(AC), shifted to Unit 6. Also, upr_C became greater than upr_D and control point, cp(CD), shifted to Unit 1 and Day 12. This change in the location of cp(CD) forced the position of the start of the D line to be one day later than before. In turn, this forward shift in the D line caused the project duration to be increased by one day. To draw on a physical analogy, it’s as though the C line kicked the foot of the D line downstream.

ROTATING MULTIPLE PRODUCTION LINES

Still further reduction in the project duration may be obtained if both the unit production rates of the C and D lines are increased. Suppose that the rate for the C line is increased to 1/2 u/d as before and that the rate for the D line is further increased to 1 u/d. Figure 23 shows this diagram.

The control point for the C line, cp(AC), is in Unit 6 at Day 18 as in Figure 22. Both activities D and E are preceded by Activity C and both now have the same unit production rate. There are two control points for each of these lines, one in Unit 6 at Day 20 and the other in Unit 4 at Day 18. These are cp_1(CD), cp_1(CE), cp_2(CD), and cp_2(CE). The figure shows the production lines for the D and E activities drawn through these points and separated by a small amount for clarity. The control points, cp(DF) and cp(EF), for the F production line are located in the usual manner in Unit 6 at Day 21, and the F line is drawn from the end of Day 16 to the finish at Day 22.

The project duration has now been reduced with this schedule to 22 days, and two controlling sequences and two critical paths have been created. All the activities on the A, C, D, E, and F lines are critical, but only the A activities in Units 1 through 6, and the C, D, E, and F activities in Unit 6 are parts of these sequences and paths; they are marked with heavy solid lines. The other critical activities are resource-critical and are marked with the heavy dashed lines.

The schedule represented in Figure 23 is a very tight schedule and would be difficult to maintain in actual construction practice. It is not the minimum schedule, as additional resources could be applied to any or all activities and unit production rates could be increased. The skill of the planner in balancing the cost of adding resources with the reduction in project duration, or in balancing the cost of interruption of resource usage with activity criticality cannot be mechanically represented. It is this skill that sets a superior planner apart from others.

INTEGRATING CPM WITH RSM

The Critical Path Method has been applied with varying degrees of success to projects where length units predominate. Sometimes its use has been successful, but most attempts to apply it, especially to highway and road work, have been less than satisfactory.

The basic cause of poor CPM performance in the highway area lies with the CPM assumption that activities can be sequenced one after another and that an activity can start only after its predecessor is finished. Yet every part of a roadway has the same set of activities, base, base course, surface, etc., and an activity can start a relatively short period after its predecessor has started. The overlapping CPM technique can create a model that is close to this actual condition, but the resulting schedule still remains unsatisfactory because the technique requires the overlap of discrete activities, when in reality, there are almost continuously overlapping activities.

The RSM method satisfies the need for continuous overlap of activities, provides for continuity of resources, and is a very accurate project model for roads and highways. However, the complete highway project may also contain bridges, retaining walls, drainage structures, and the like, which do have discrete activities and should be modeled using the CPM method. The following example will illustrate one technique for the integration of CPM and RSM in the same diagram.

Assume that it has been decided to upgrade a 6000 foot stretch of a two-lane road traversing a gently rolling area. The profile of the road shows a hill on the west 1500 feet of the stretch with a four percent grade both up and down. A center 1500 foot portion of the stretch is depressed where the road crosses a small stream passing through a 50 inch diameter Corrugated Metal Pipe. The grade in this stretch is two percent down to the stream and three percent up toward the east. The hill followed by the sag in the roadway appears to be a cause of several accidents.

The road surface along the entire 6000 feet is in poor condition because of inadequate drainage. The old base course was too thin and had become plugged with silt over the years. Also, flood waters have tended to pond upstream of the culvert indicating the need for greater culvert capacity. To remedy these ills, it has been decided to replace the CMP culvert with a concrete box culvert about eight feet high, nine feet wide, and eighty feet long. Clearly, the construction of the culvert will be a major part of the total project. In addition, the road grade will be flattened, the base material will be replaced, and the road will be resurfaced.

Two questions immediately arise: (a) How long is it going to take?, and (b) What is a reasonable plan for doing the work? Both questions may be answered by the preparation of a RSM schedule that includes all parts of the project.

Because the culvert is a major element of the project, attention must be focused there first. Figure 24 is a CPM diagram of the culvert construction where it is assumed that the culvert will be divided into three parts, a north section, a center section, and a south section. To minimize the time, the plan is to construct the center section first, followed simultaneously by the north and south sections. Although forms for one section might be reused on another section, the erection of north and south forms together resolves a tradeoff between the extra cost of the formwork and the cost and time of project completion. In the figure, all the network activity durations are given in working days and the computations show that the culvert can be completed in 49 days.

Figure 25 shows a RSM diagram which incorporates the CPM schedule of the culvert. This diagram is plotted with the time in working days on the Y-axis and the centerline stations on the X-axis, with the stations measured from the west end of the project.

The plot begins at Sta. 25+00 with the removal of the existing surface and base. Assume this removal is done at a unit production rate of 5 stations per day (5 Sta/d). Work progresses eastward from Sta. 25+00 and ends at Sta. 60+00 on Day 7. Equipment and personnel are then shifted back to Sta. 25+00 and removal proceeds to Sta. 0+00, where all removal is scheduled to finish on Day 12. This plan keeps the removal crews continuously employed and allows excavation for the culvert to begin on the second day.

The vertical double line at Sta. 30+00 represents the culvert diagrammed in Figure 24. It is scheduled for completion on day 50 (49 days for the culvert plus one day for surface and base removal). As backfilling of the culvert can begin at the end of Day 40, the process of excavation, transportation, and backfill begins at that time. Hill excavation and road backfill might begin earlier, while the culvert was under construction, but this would mean that excavation equipment and personnel would be forced to wait for the finish of the culvert before completing the culvert backfill. A seemingly better plan is to delay the start of the hill excavation and do both the road backfill and the culvert backfill at the same time.

It will be noted that the excavation and backfill are not shown as activity production lines. Instead, they are shown as rectangles whose base is distance and height is time. These activities are more like CPM activities because excavation begins at the top and works to the bottom of the cut with backfill proceeding in a reverse manner. There are surely production lines for both the hill excavation and backfill, but detailed plotting of these production lines only would clutter the diagram with unneeded information.

Once excavation and backfill is complete, finish grading can begin. The choice here is to begin finish grading at Sta. 35+00 and progress to the end of the project at a unit production rate of 10 Sta/d. The crews return to Sta. 35+00 and work back to Sta. 0+00 at the same rate, allowing scheduled completion of this activity to be Day 56.

After completion of the finish grading, the placement of the asphalt base course can begin. Note that the choice is to begin at Sta. 0+00 and work at a unit production rate of 7.5 Sta/d until the end of the project at Sta. 60+00. The base course is therefore scheduled to begin at the end of Day 56 and end on Day 64.

The bituminous concrete surface is designed to have two layers. The placement of each course is estimated to proceed at a unit production rate of 24 Sta./d which is considerably faster than the estimate for the base course. It may be assumed that the same equipment is to be used for all asphalt courses. Therefore, the activity, “Base Course” must be complete before “Bituminous Course #1” can start. Similarly, “Bituminous Course #1” must be complete before “Bituminous Course #2” can start. Using this reasoning, the production lines for these two bituminous surface activities set a schedule from Day 64 to Day 66.5 for “Bituminous Course #1” and from Day 66.5 t o Day 69 for “Bituminous Course #2.”

Guardrail needs to be installed between Sta. 20+00 and Sta. 35+00 because the grade has been raised in this section. Guardrail erection can begin after “Bituminous Course #2” has been laid, and a decision is made to start the installation of the guardrail at Day 68 because “Bituminous Course #2” is scheduled to be completed at 67.3 days. At a unit production rate of 5 Sta./d, the guardrail activity is scheduled to finish at Day 71.

Seeding and mulching of the raw slopes is the last activity included in this project. It logically must follow the laying of the bituminous surface and the installation of the guardrail. The unit production rate is estimated at 10 Sta./d. The seeding activity must not overtake the installation of the guardrail, so the decision is made to start the seeding activity at Sta. 0+00 on Day 68, which allows about one half day float time at Sta. 35+00 between the guardrail and seeding activities. The finish of the seeding and mulching activity on Day 74 marks the end of the project.

The above example provides a reasonable schedule for the project and establishes the project duration as 74 days. It also demonstrates the integration of CPM and RSM for a road upgrade. The project activities have been simplified and some items have been omitted in the interest of clarity, however, the diagram illustrates the main features of the process. Actual activities, quantities, unit production rates, and activity durations must be used in developing a specific schedule for an actual road or highway project.

CONCLUDING REMARKS

Construction contractors often need to model multi-unit projects wherein activities repeat from unit to unit. These activities need to be scheduled so that their required resources are continuously used once they arrive on the construction site. The use of typical CPM scheduling techniques cannot ensure this continuity in resource utilization because only technical precedence and resource availability constraints are shown in CPM networks. The Repetitive Scheduling Method (RSM) recognizes the additional resource continuity constraint that cannot be shown in a network, and thus provides for continuous resource usage.

RSM is presented here as a scheduling method that simplifies and generalizes several variously named, multi-unit scheduling techniques that have been cited in past publications. It incorporates commonly accepted activity precedence concepts from CPM, and can be applied to both vertical and horizontal projects that may contain either discrete or continuous activities.

An RSM schedule is presented graphically as an X-Y plot of unit production lines that continue across designated units of the project. One axis of the plot represents units and the other time, and the repetitive units may be assigned to either axis, the particular assignment being chosen for convenience and to clearly communicate the schedule information. Typically, a resource production line appears in the diagram as a continuous straight line. However, some activity segments of the line may have different slopes if the work content in repeating units is not uniform.

The construction of RSM schedules involves the positioning of successive unit production lines by using the new concept of control points. As shown earlier, there is a specific point along each production line that controls the schedule position of its successor production line. This point, called a control point, tends to be located toward the first unit in the sequence of units if the lines diverge, and toward the last unit in the sequence if the lines converge. These control points have significance in the determination of the project duration, and serve as points of rotation for unit production lines whose resource rates are increased or decreased.

RSM also introduces a new concept for the determination of the project duration. As with all projects, the duration must be determined by some sequence of activities that extends from project start to project finish. This sequence in RSM is called the controlling sequence and includes the activities of the first production line from project start until the first control point is reached. It then switches to the next production line and includes all activities on that line until the next control point is found. The sequence continues to include activities in this fashion, switching from production line to production line at control points, until reaching the end of the project. An RSM controlling sequence may include both critical and non critical activities. Conversely, activities can be critical because of resource continuity (resource critical), and thus not be part of the controlling sequence.

The unit production rate of any activity can be increased or decreased by altering the composition of the crews or equipment needed to carry out the activity. This causes the associated unit production line to rotate about a control point and to increase or decrease the project duration. However, care must be taken in choosing the activity and resource to change; a poor choice may shift the location of the controlling point for the production line and result in an unexpected project length. Shortening the duration of an activity may end up increasing the duration of the project.

Because RSM schedules are based on precedence CPM scheduling techniques, it is easy to combine both CPM and RSM. Single schedules can be prepared that use RSM techniques to model a project’s repetitive activities, and that use conventional CPM techniques to model the project’s discrete nonrepetitive activities. Such integration leads to more accurate scheduling representation.

RSM is a practical scheduling methodology. It uses customary work methods and crews to define repetitive activities that can be arranged in any desired pattern. RSM diagrams are easy to prepare and understand, and the unique concepts of control points and controlling sequence are quickly comprehended. The project duration, along with the start time, the finish time, and the critical status of each activity are quickly found from the diagram. Thus, RSM has all the necessary performance characteristics to serve as a convenient and practical tool for scheduling multi-unit projects.

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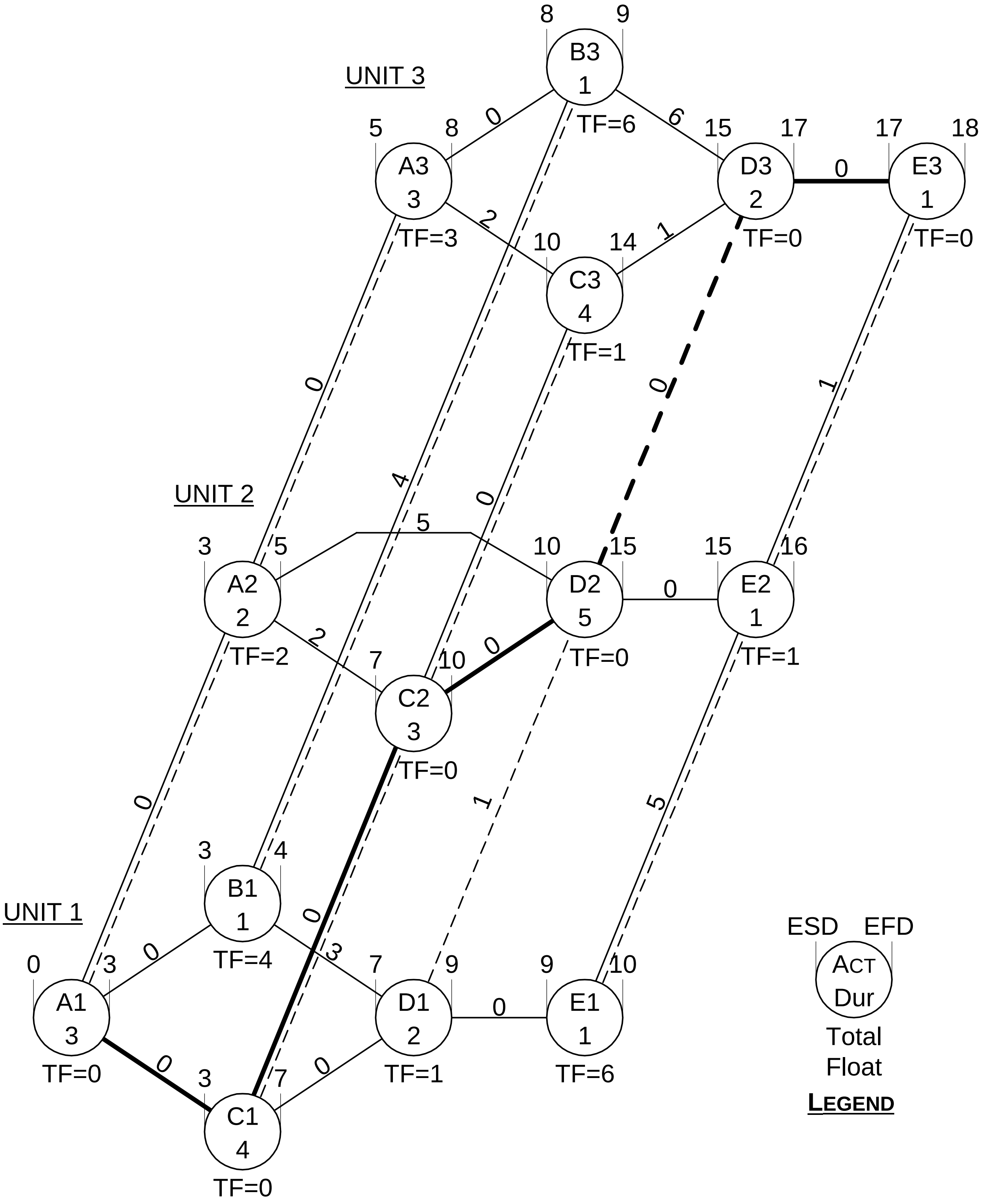
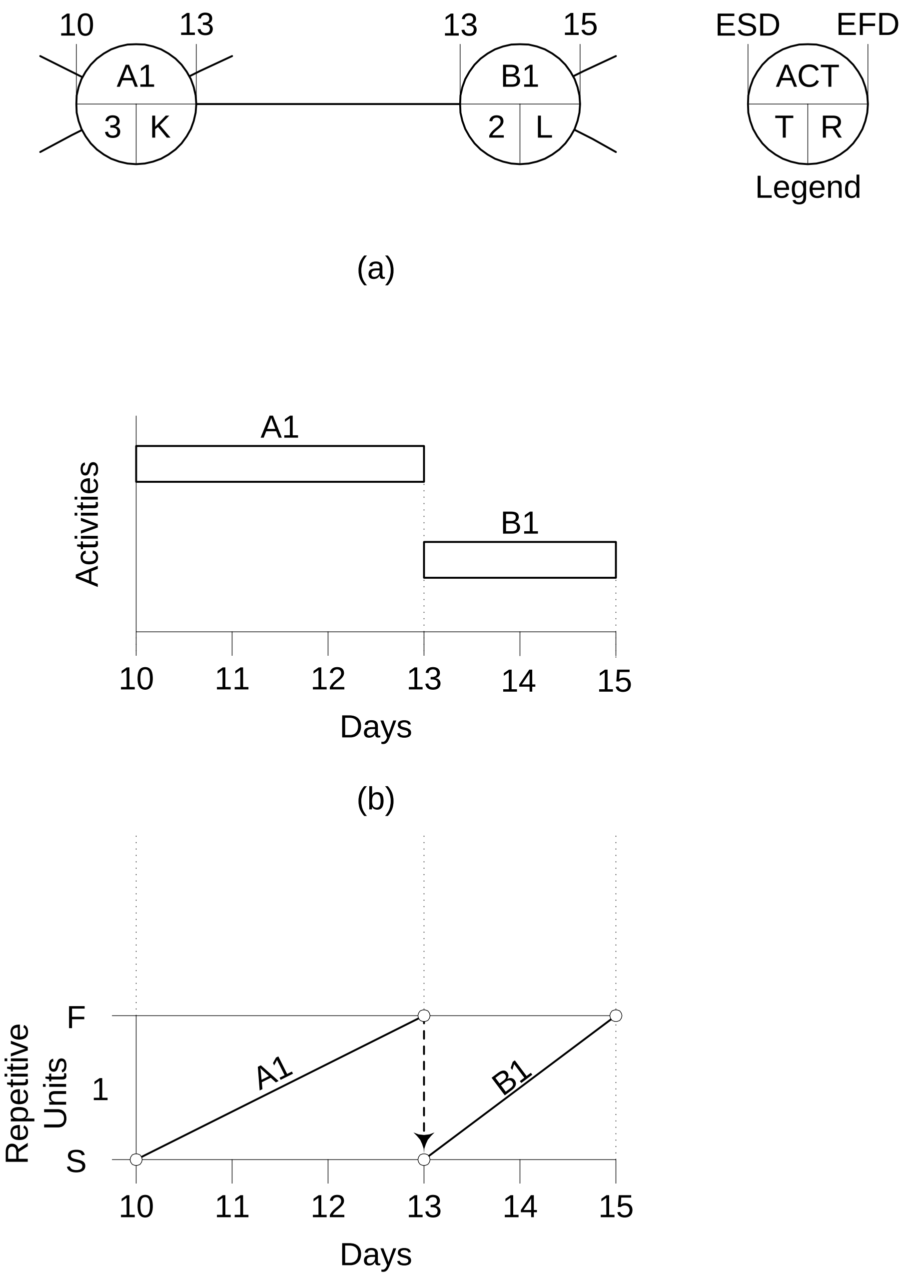


FIGURE 1
CPM NETWORK FOR THREE REPETITIVE UNITS

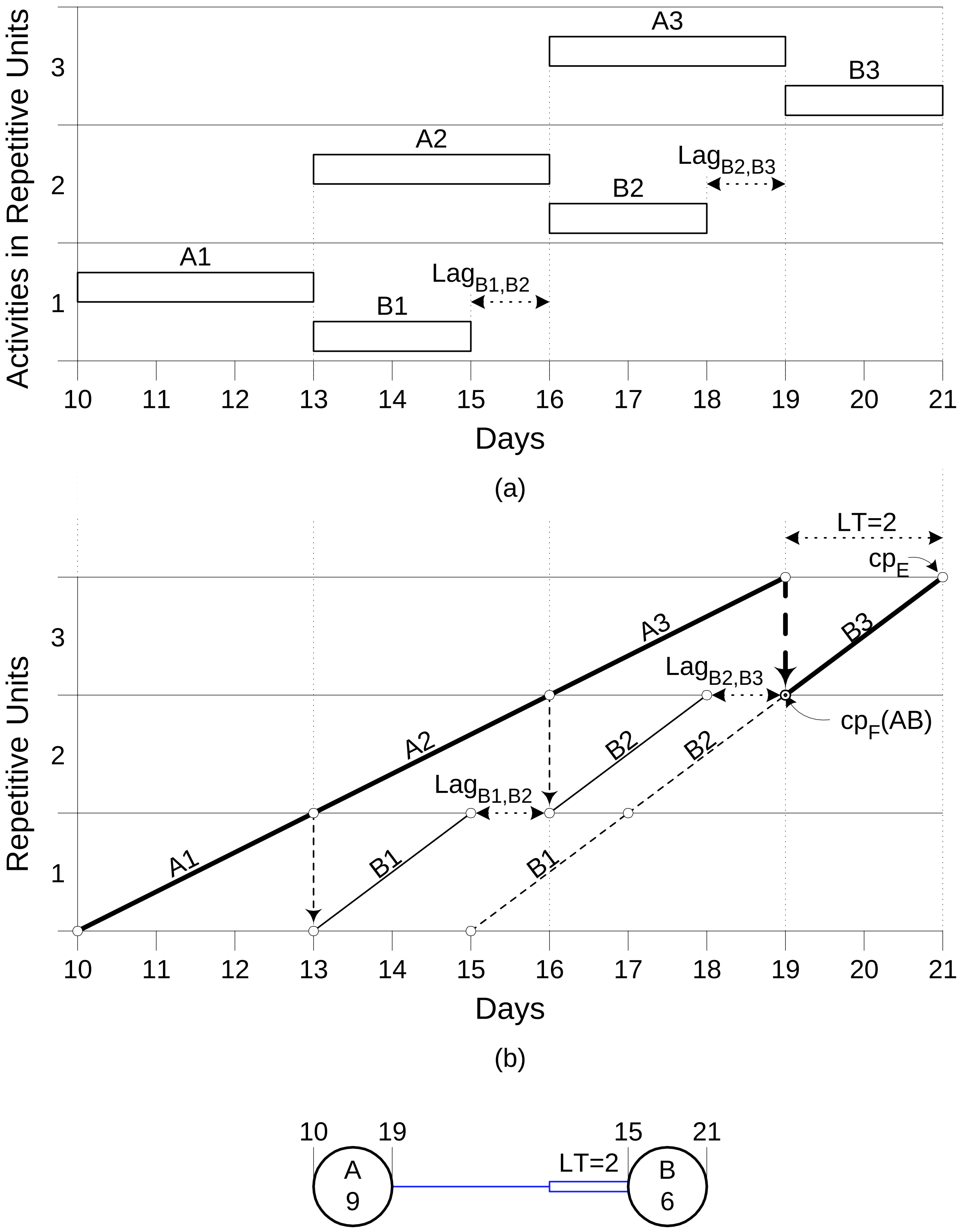


(a)

(b)

(c)

Figure 2
Converging FTS Activities in RSM



(a)

(b)

(c)

FIGURE 3

BAR CHART AND RSM DIAGRAM FOR THREE UNITS WITH CONVERGING FTS ACTIVITIES

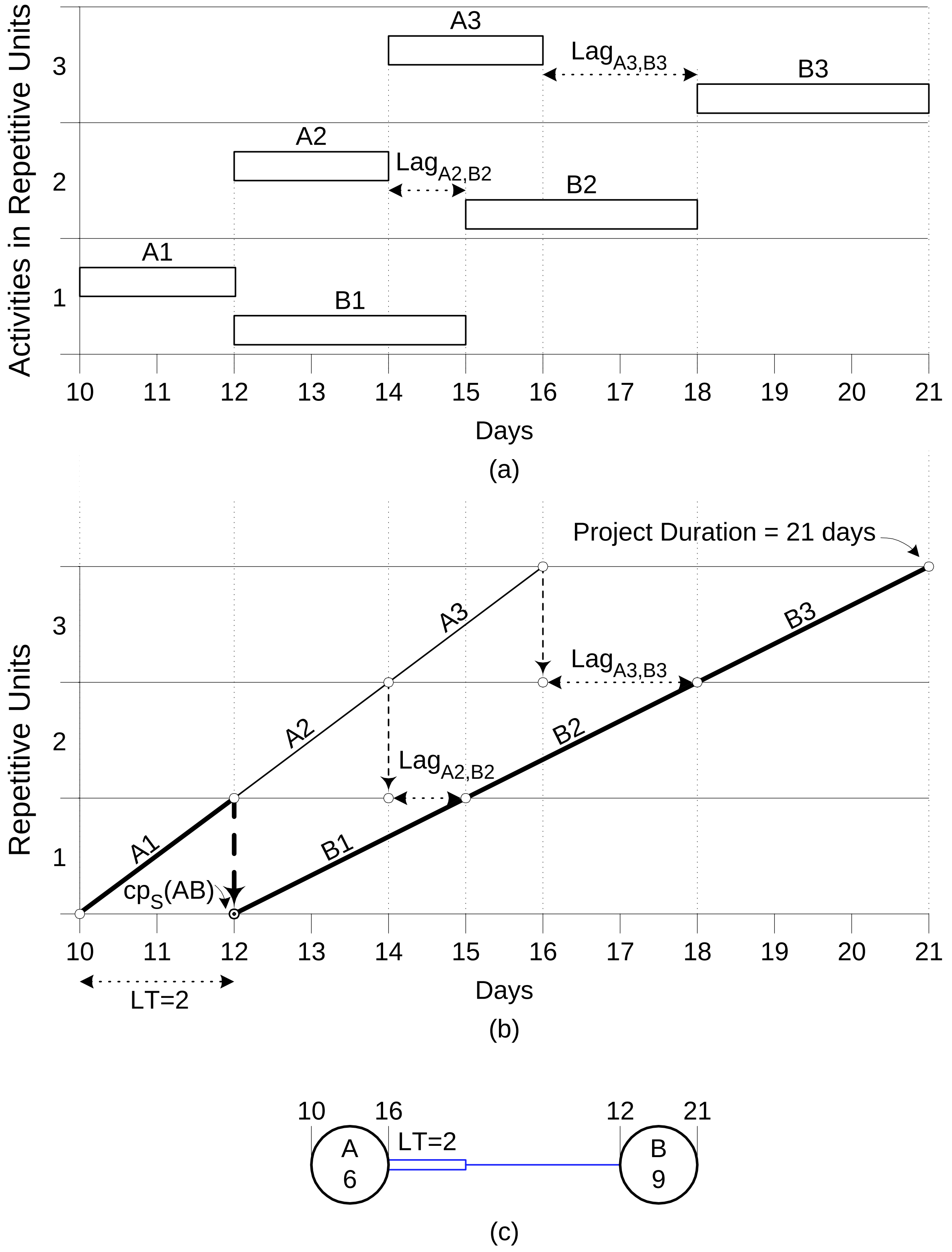


FIGURE 4
BAR CHART AND RSM DIAGRAM FOR THREE UNITS WITH DIVERGING FTS ACTIVITIES

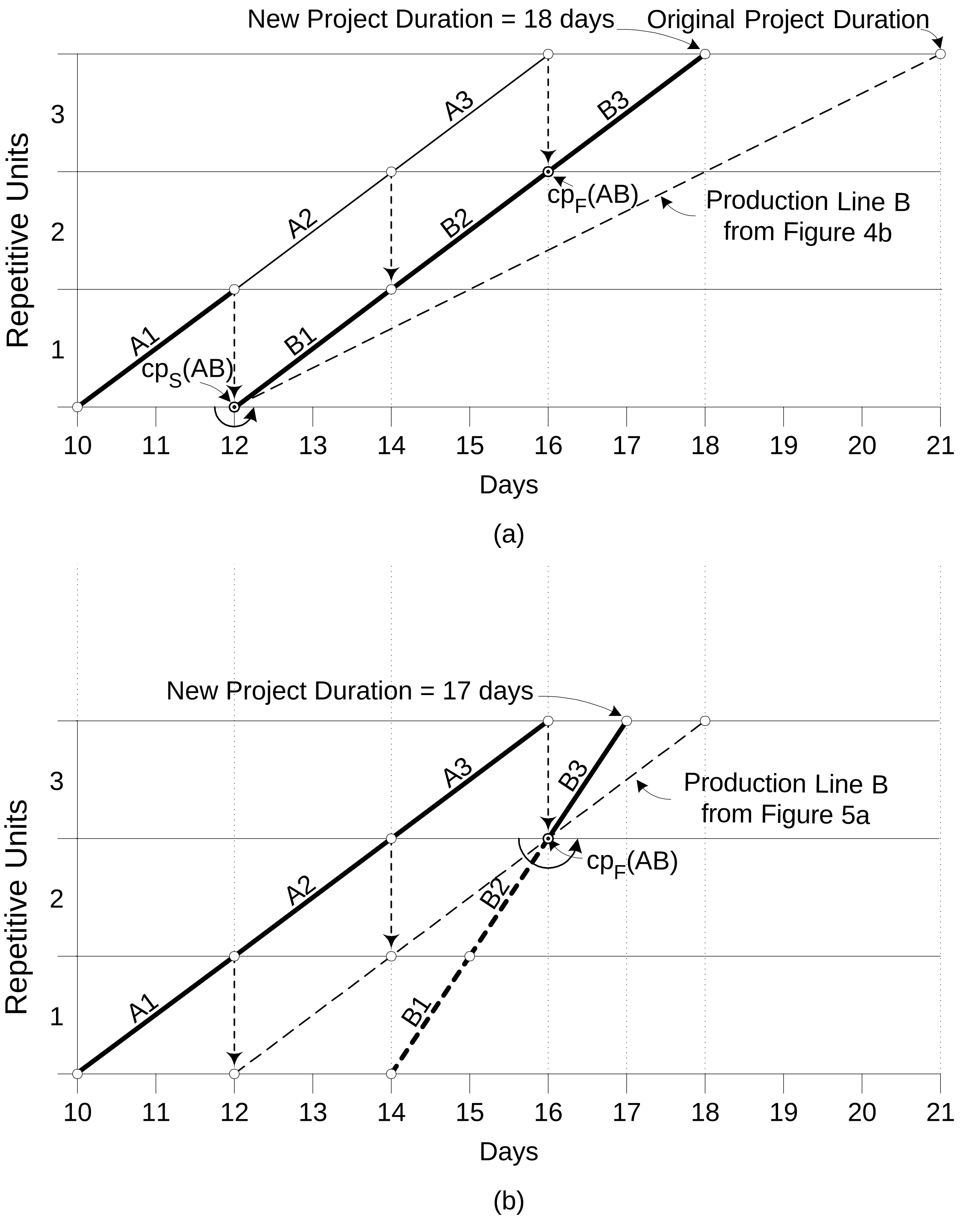


FIGURE 5
EFFECTS OF INCREASING UNIT PRODUCTION RATES
IN RSM DIAGRAMS WITH FTS ACTIVITIES

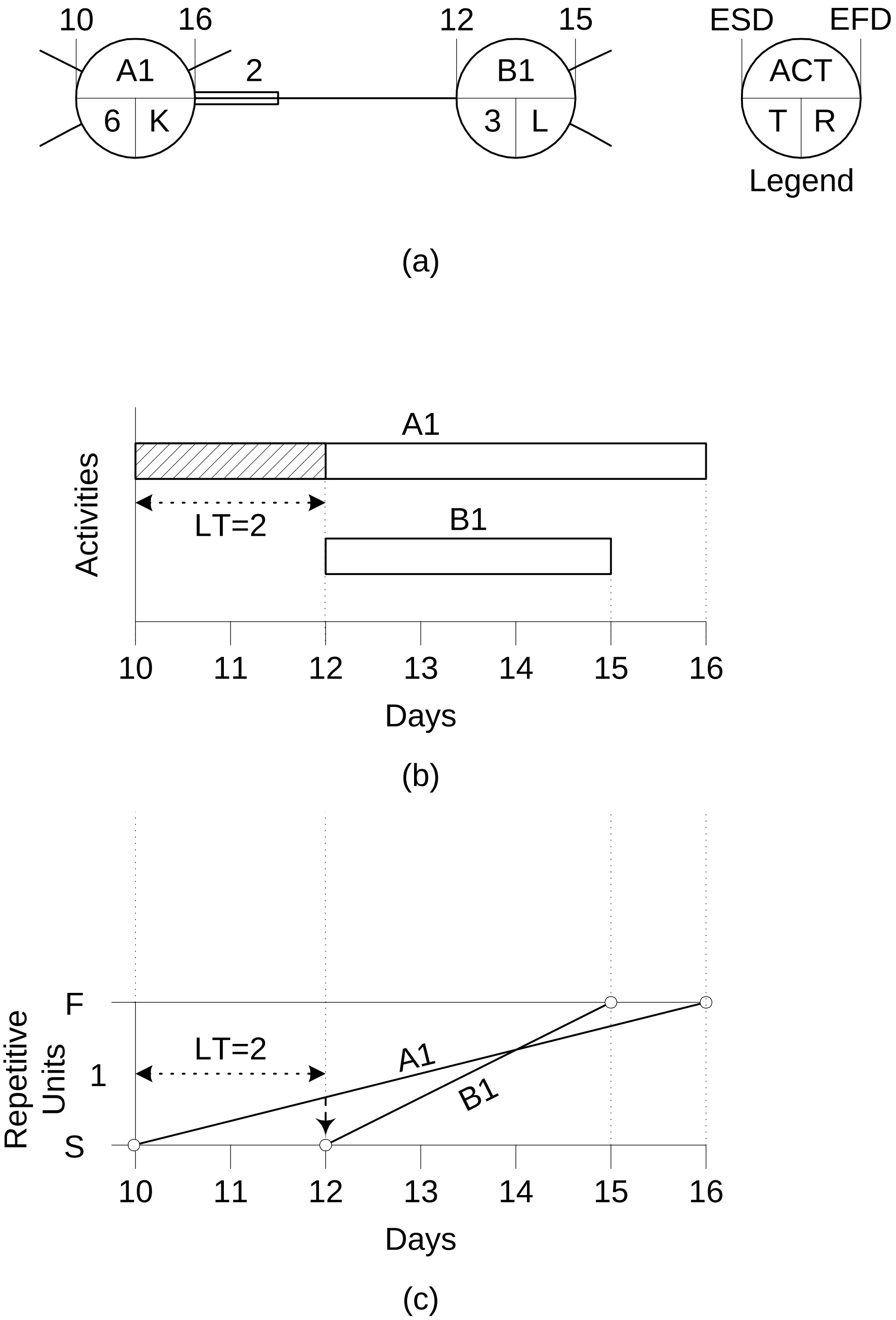
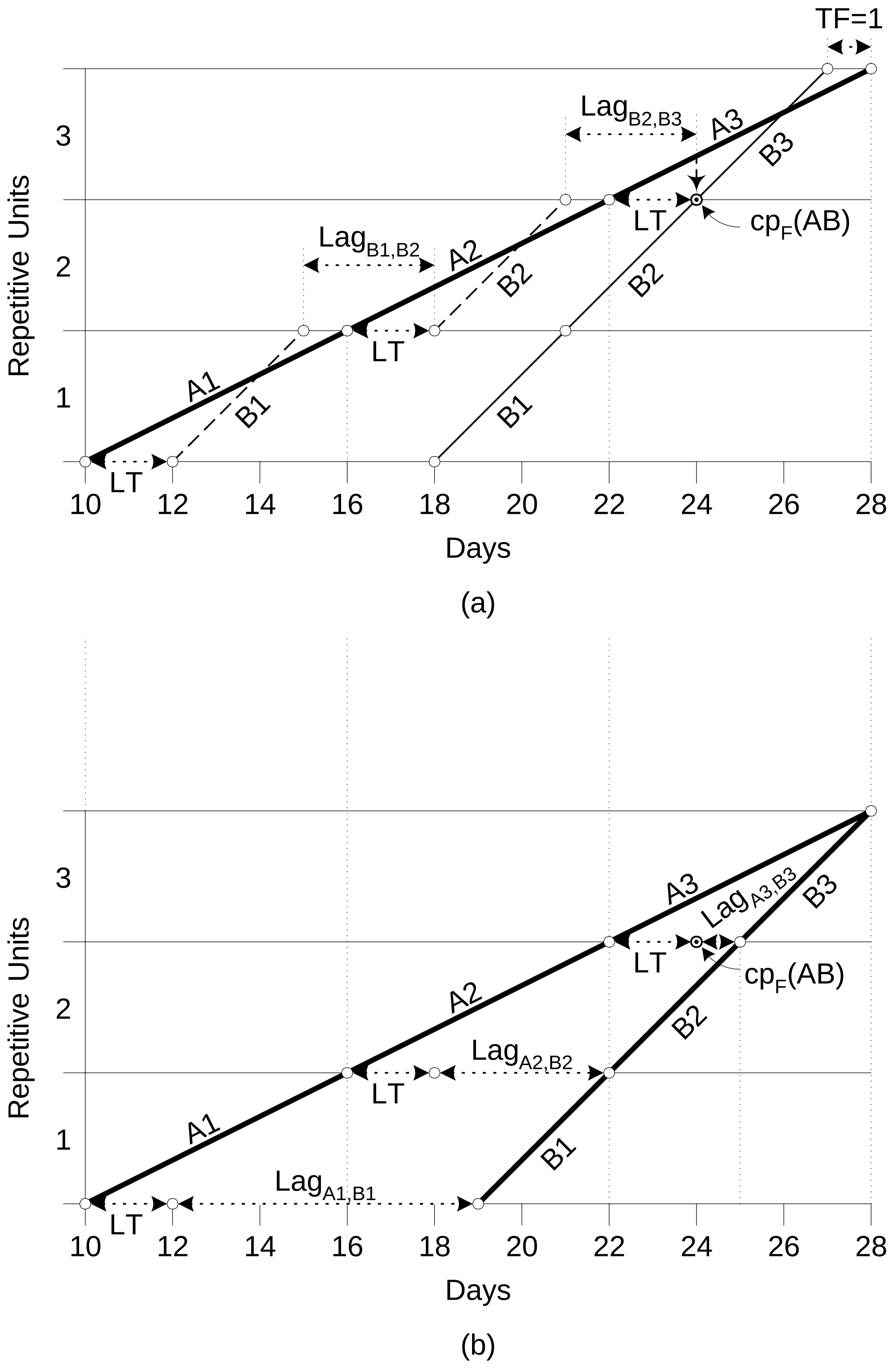


FIGURE 6
CONVERGING STS ACTIVITIES IN RSM



(a)

(b)

FIGURE 7
RSM DIAGRAM FOR THREE UNITS WITH CONVERGING STS ACTIVITIES

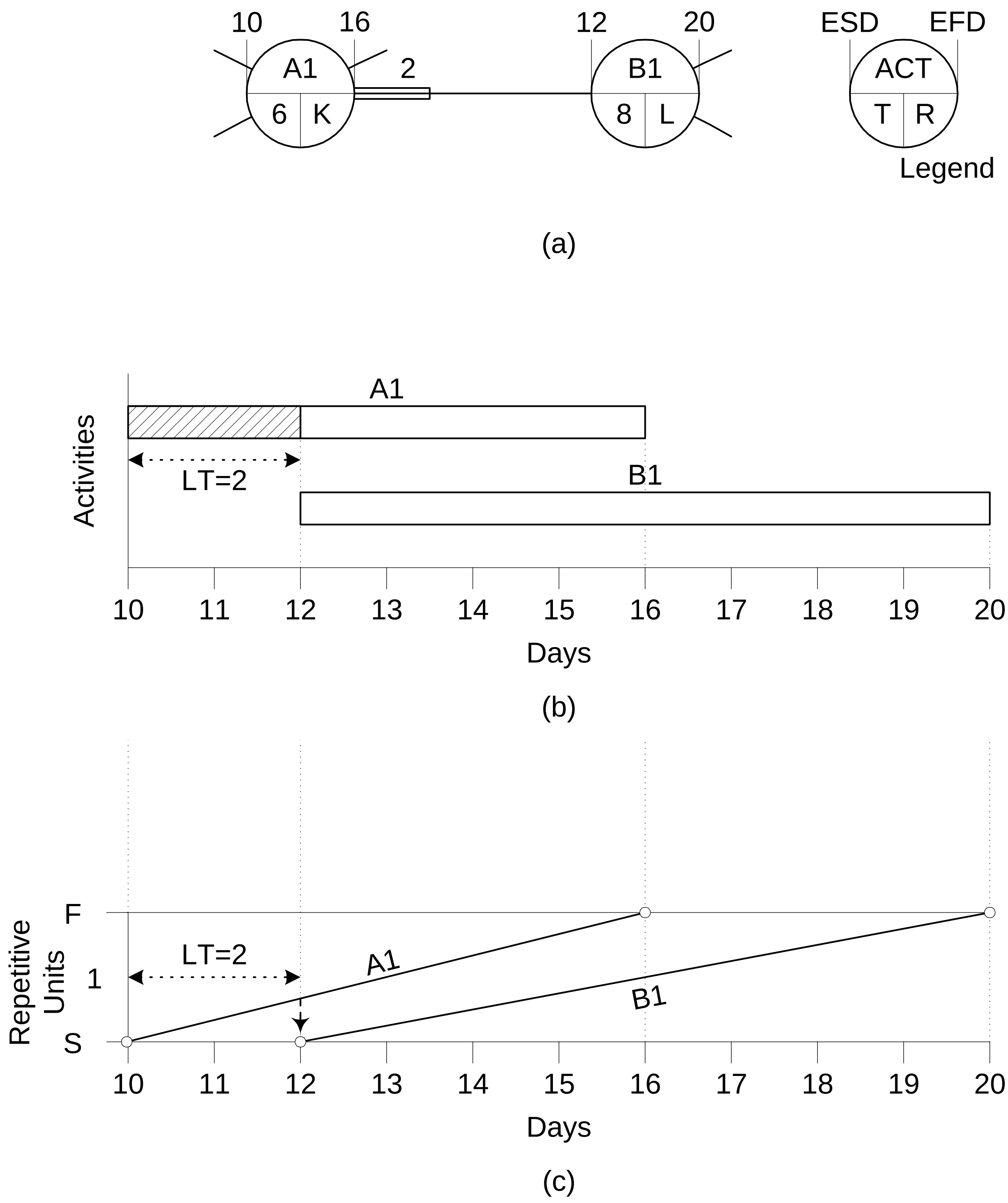


FIGURE 8
DIVERGING STS ACTIVITIES IN RSM

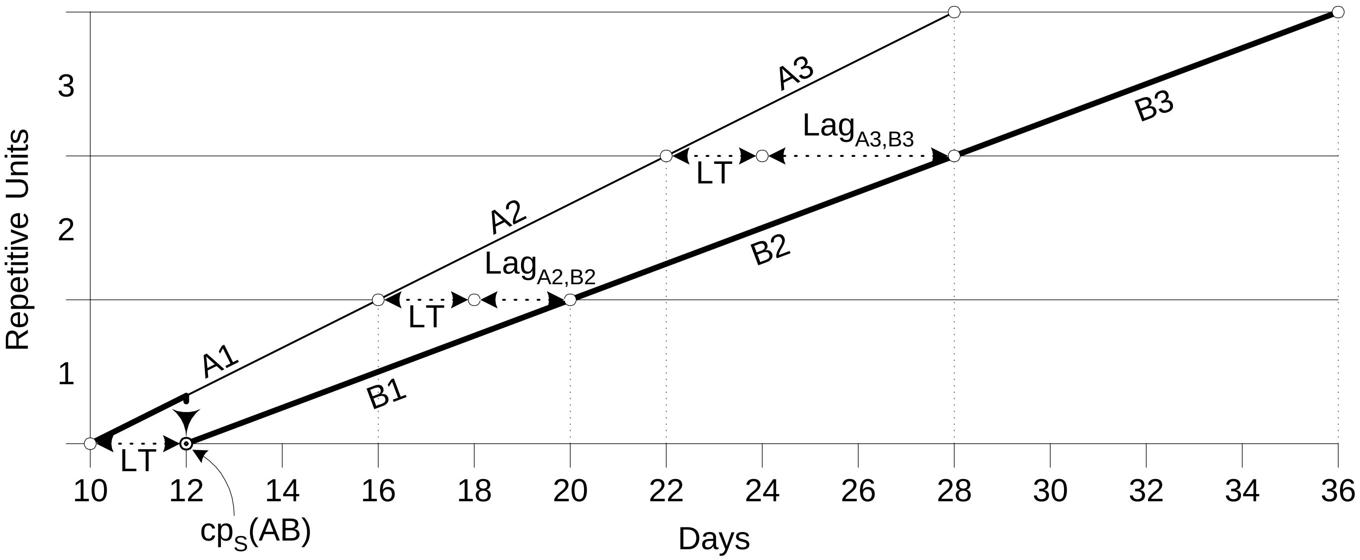
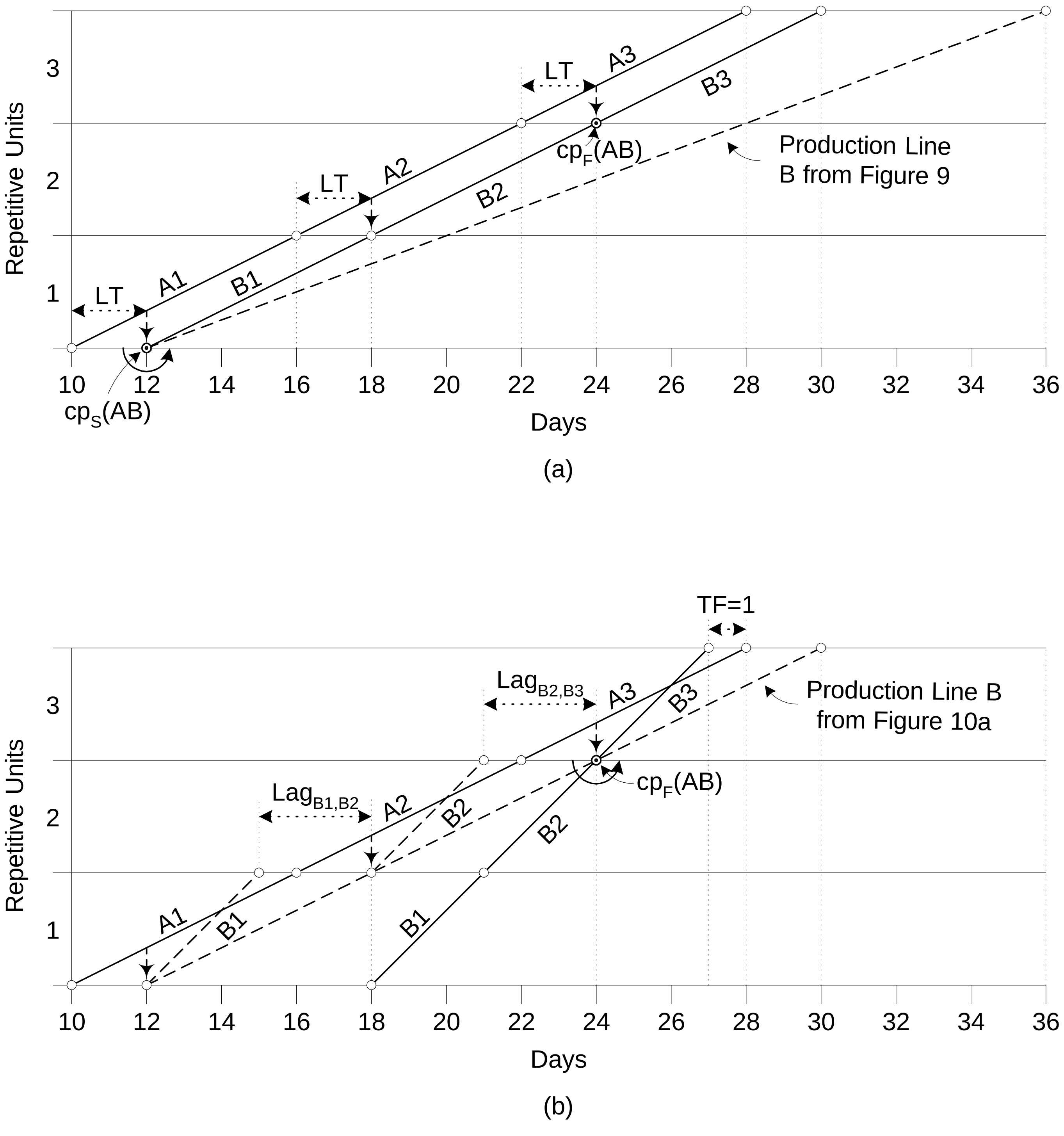
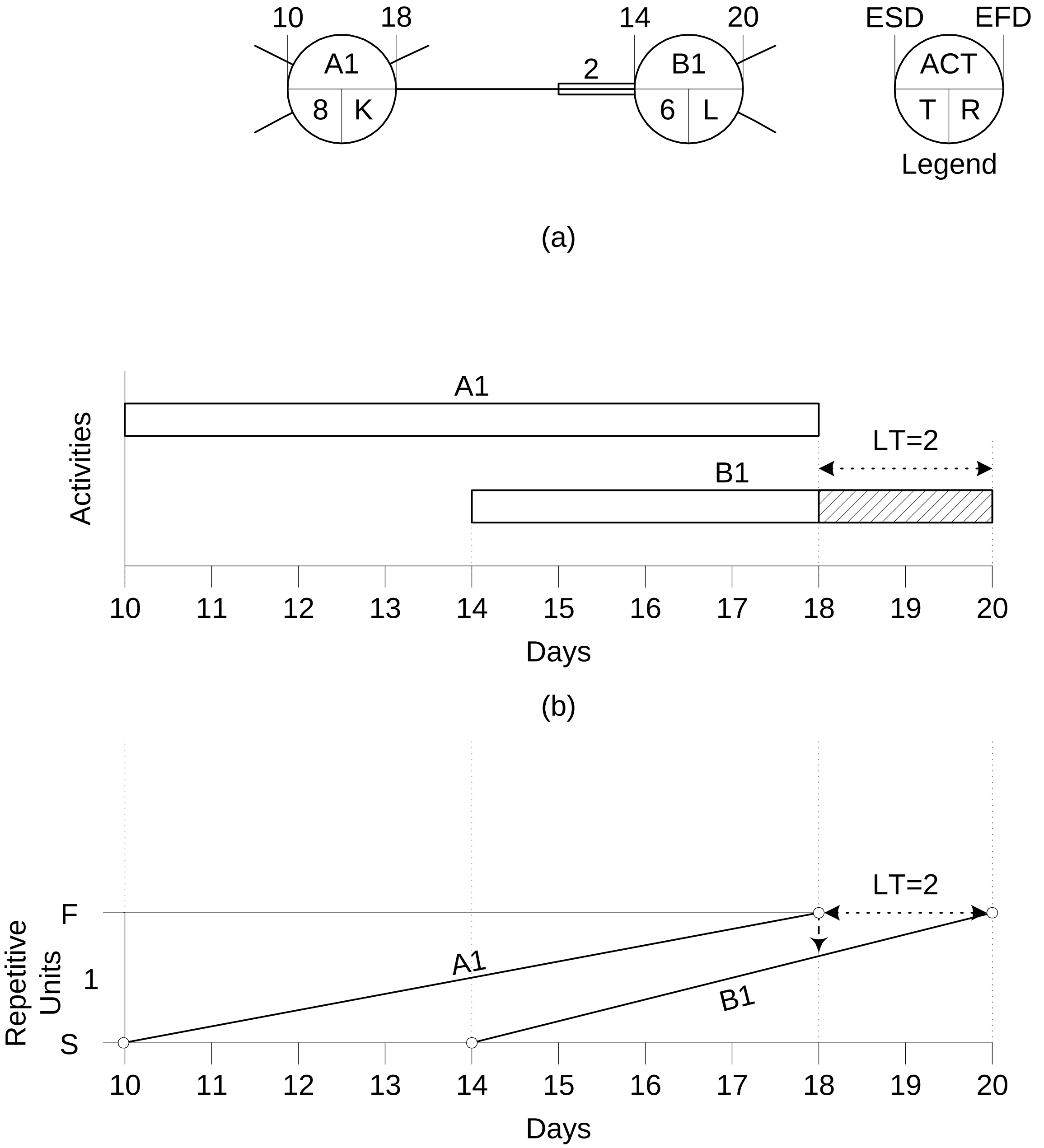


FIGURE 9
RSM DIAGRAM FOR THREE UNITS WITH DIVERGING STS ACTIVITIES


(a)

(b)

FIGURE 10
EFFECTS OF INCREASING UNIT PRODUCTION RATES
IN RSM DIAGRAMSWITH STS ACTIVITIES



(a)

(b)

(c)

FIGURE 11
CONVERGING FTF ACTIVITIES IN RSM

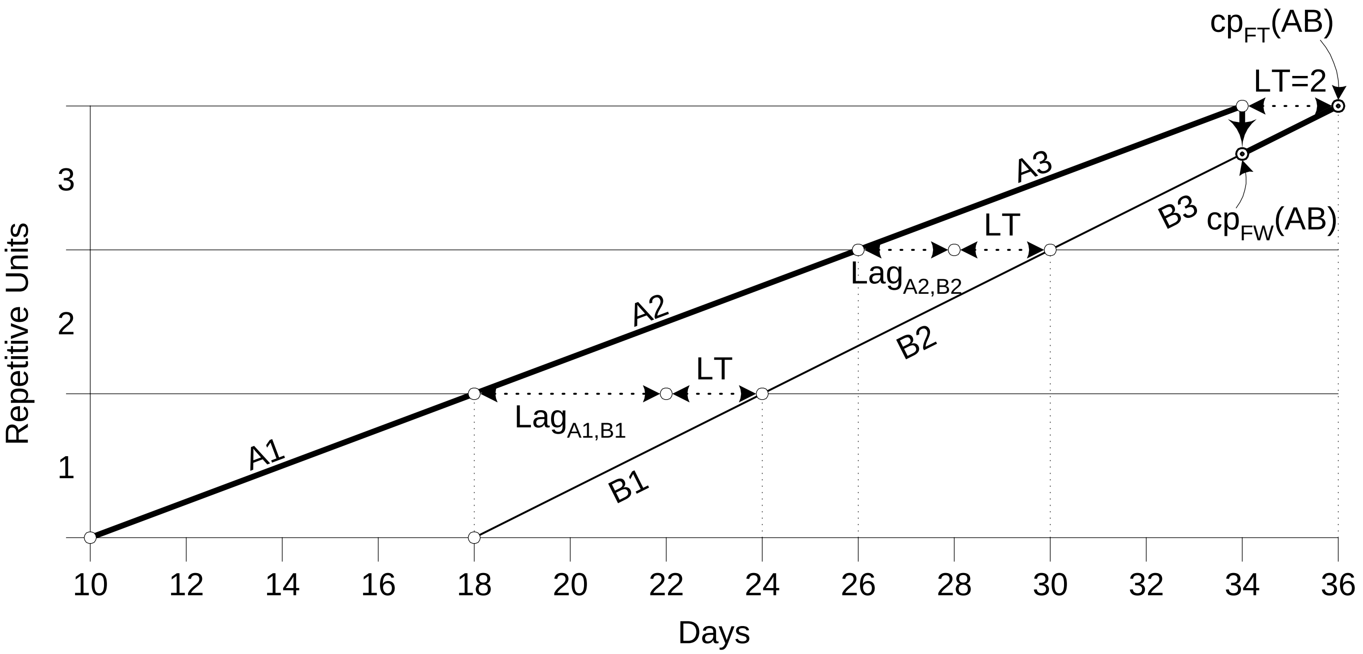
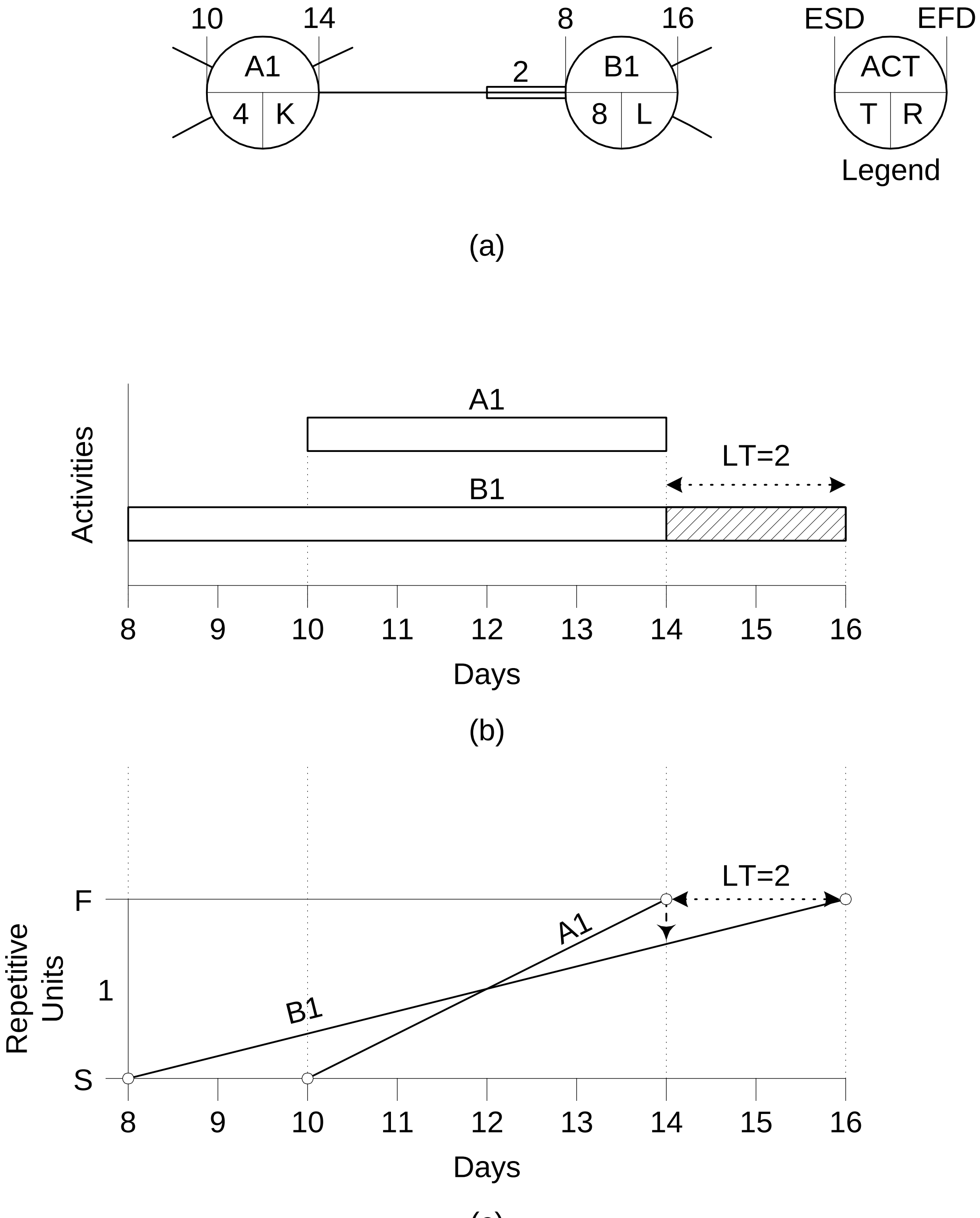


FIGURE 12
RSM DIAGRAM FOR THREE UNITS WITH CONVERGING FTF ACTIVITIES



(a)

(b)

(c)

FIGURE 13
DIVERGING FTF ACTIVITIES IN RSM

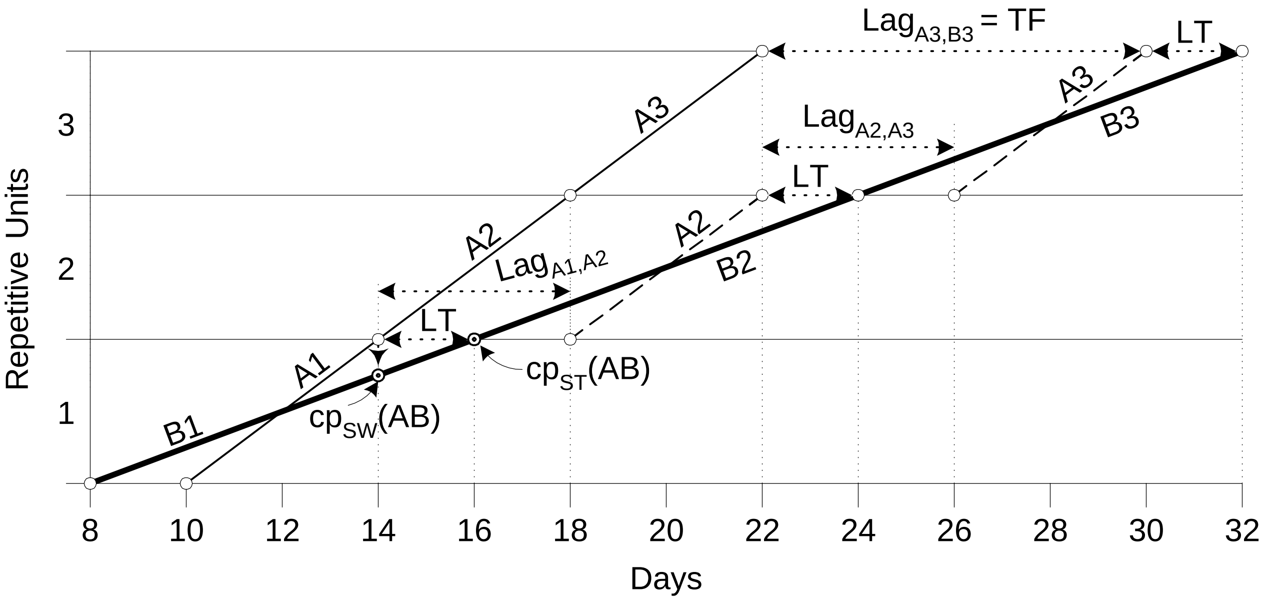


FIGURE 14
RSM DIAGRAM FOR THREE UNITS WITH DIVERGING FTF ACTIVITIES

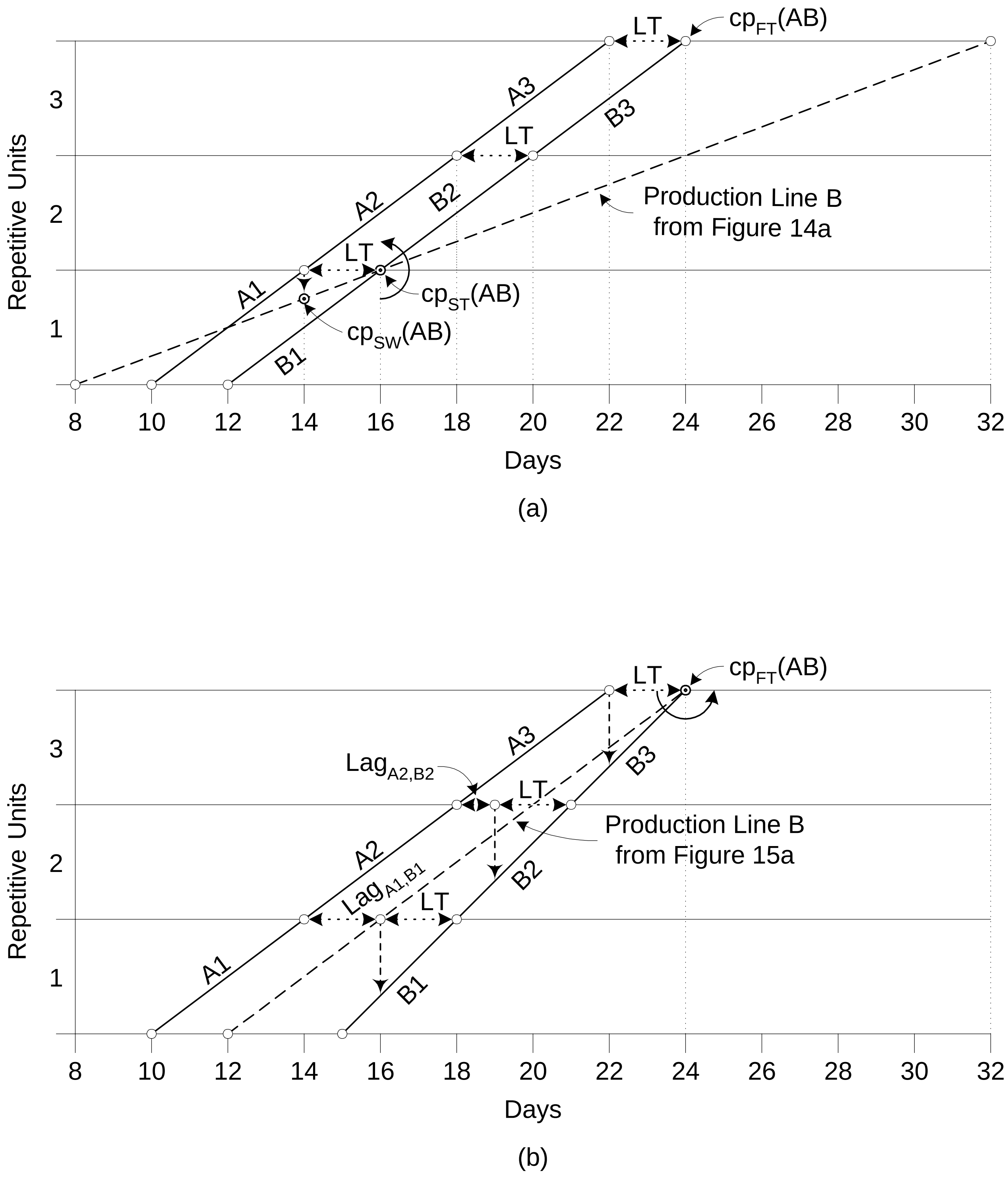


FIGURE 15
EFFECTS OF INCREASING UNIT PRODUCTION RATES
IN RSM DIAGRAMS WITH FTF ACTIVITIES

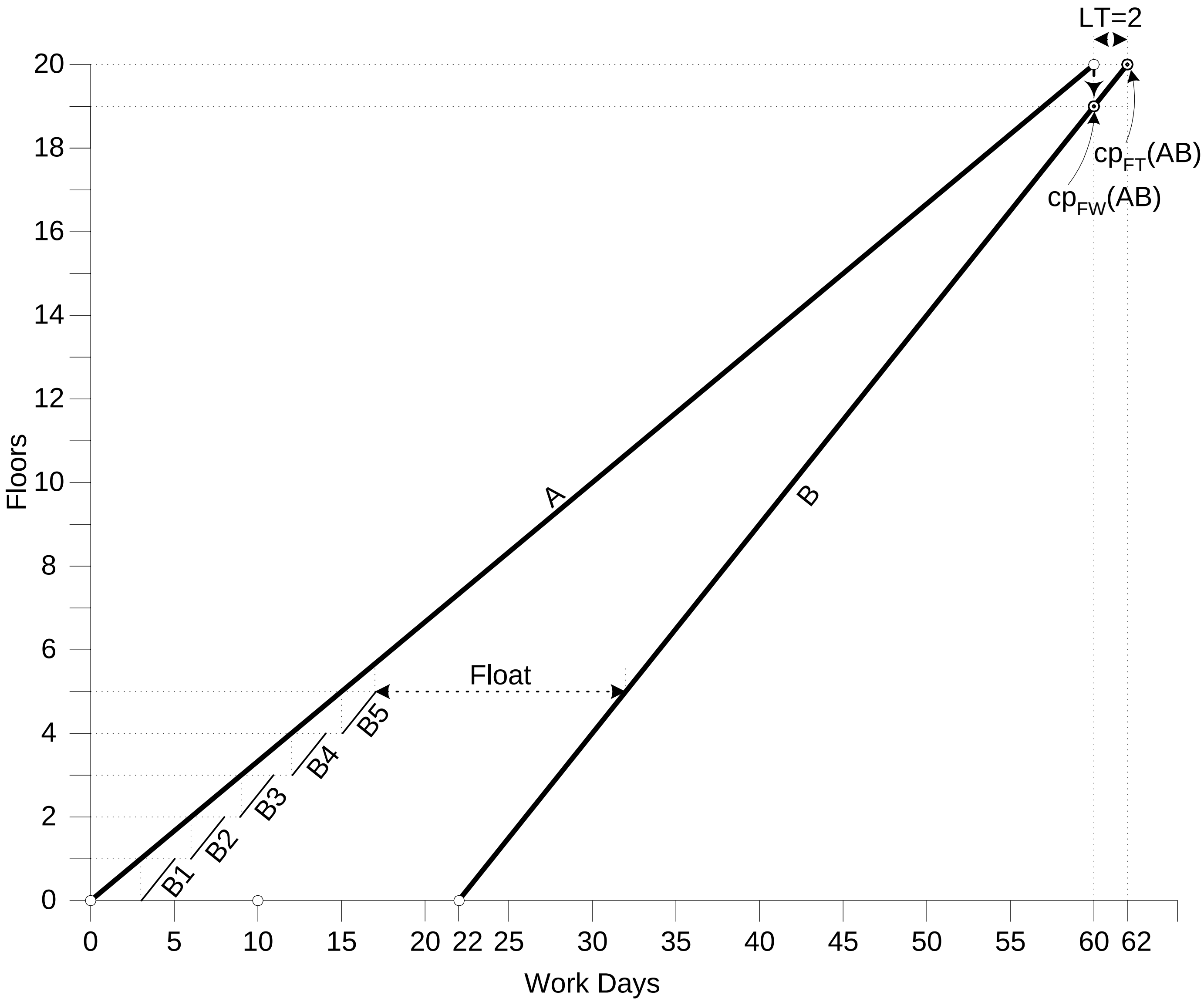


FIGURE 16
RSM DIAGRAM FOR TWENTY FLOORS WITH FTS ACTIVITIES

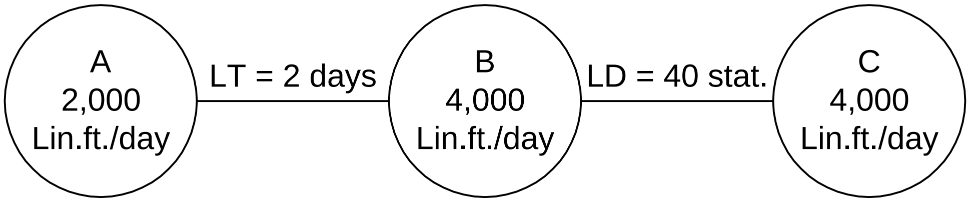


FIGURE 17
LOGIC RELATIONSHIPS FOR EVERY UNIT OF A HIGHWAY

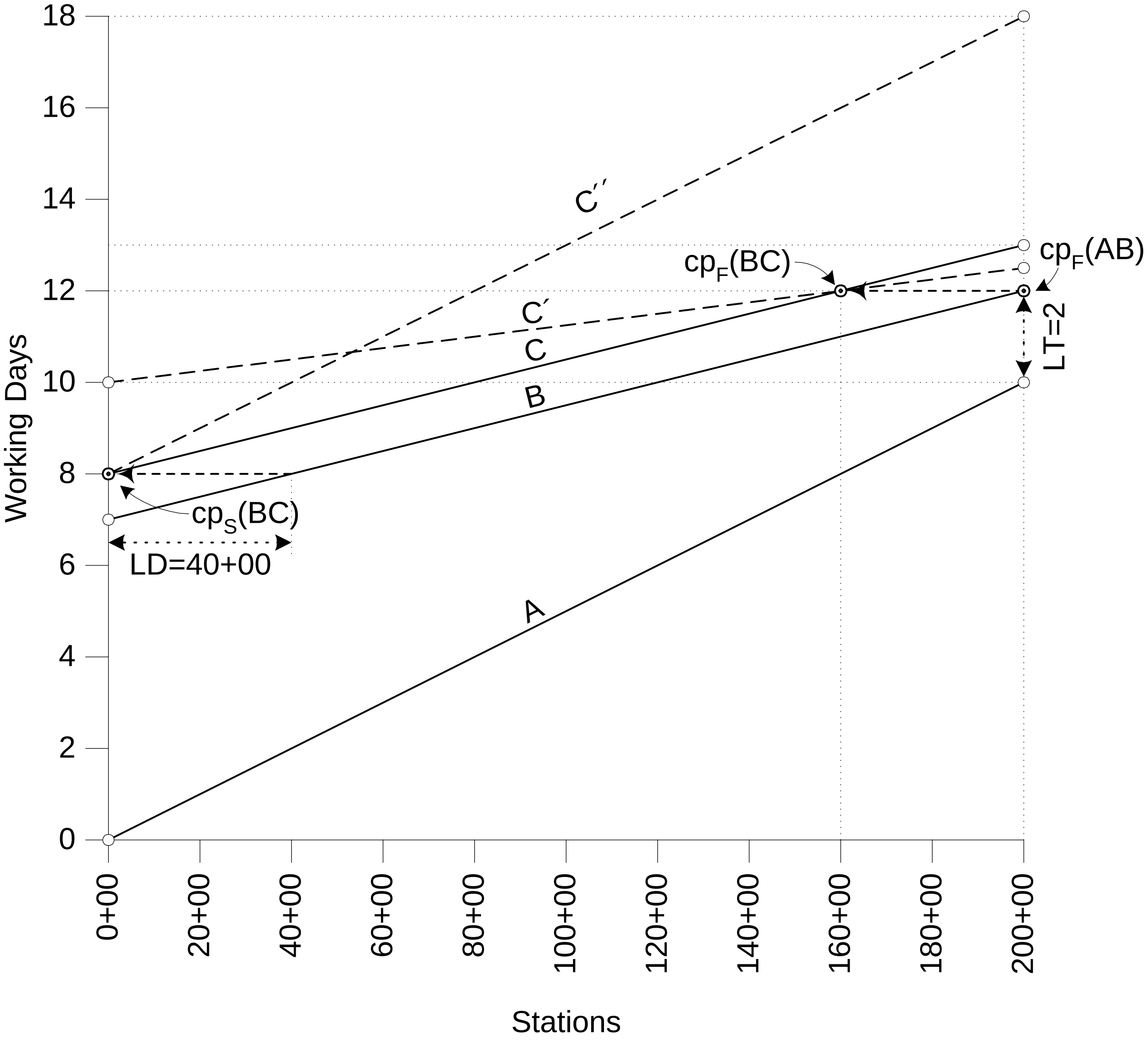


FIGURE 18
PARTIAL RSM DIAGRAM FOR A HIGHWAY PROJECT

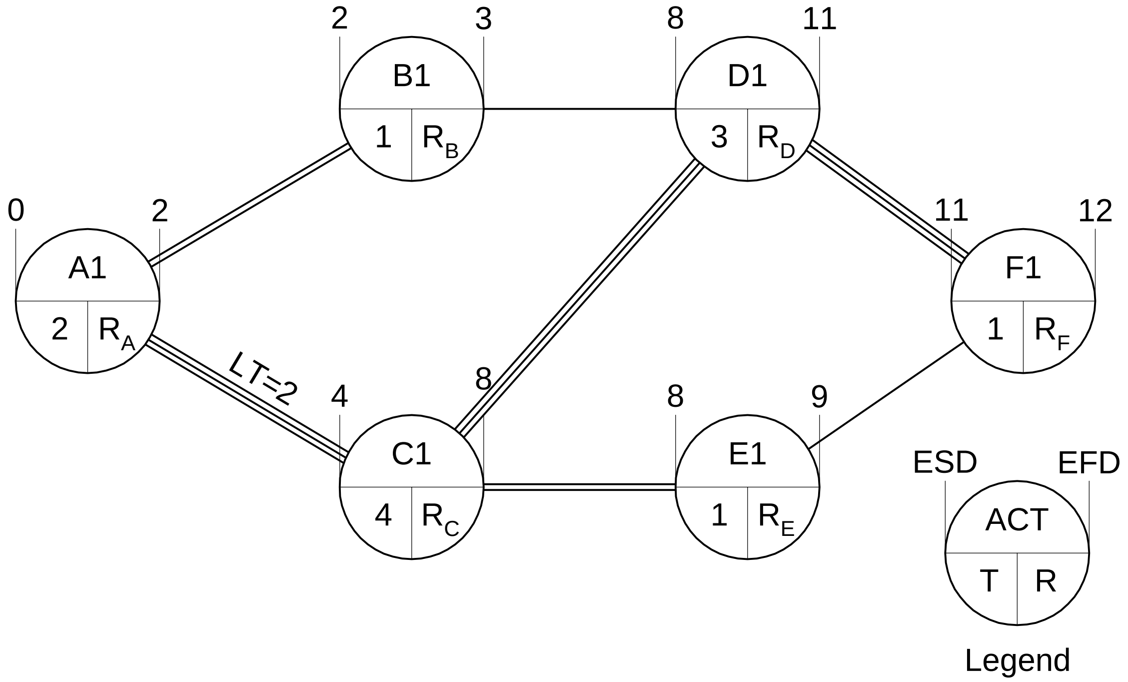


Figure 19
Precedence Diagram for Unit One of a Six Unit Project

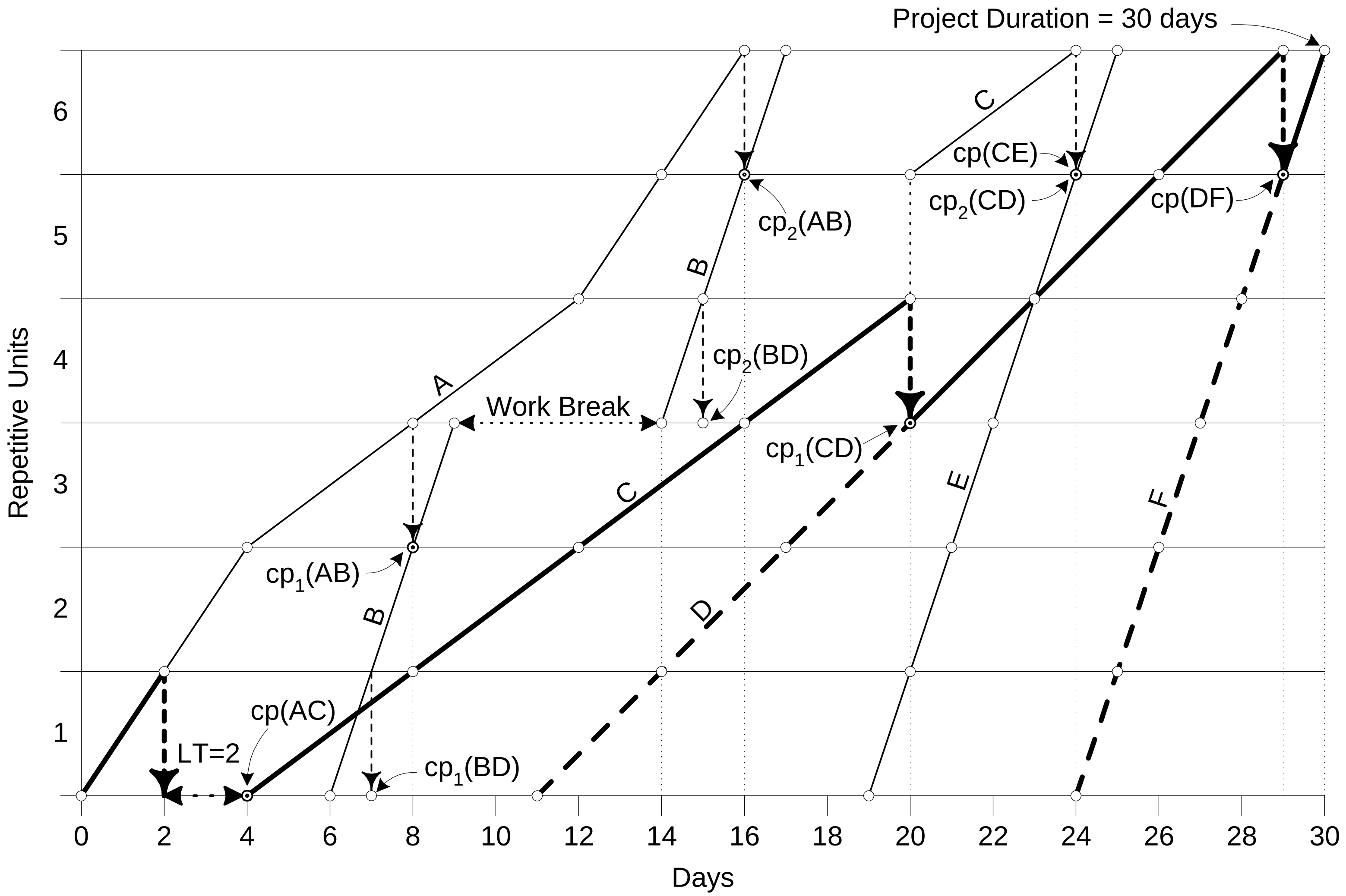


FIGURE 20
RSM DIAGRAM FOR A SIX UNIT PROJECT

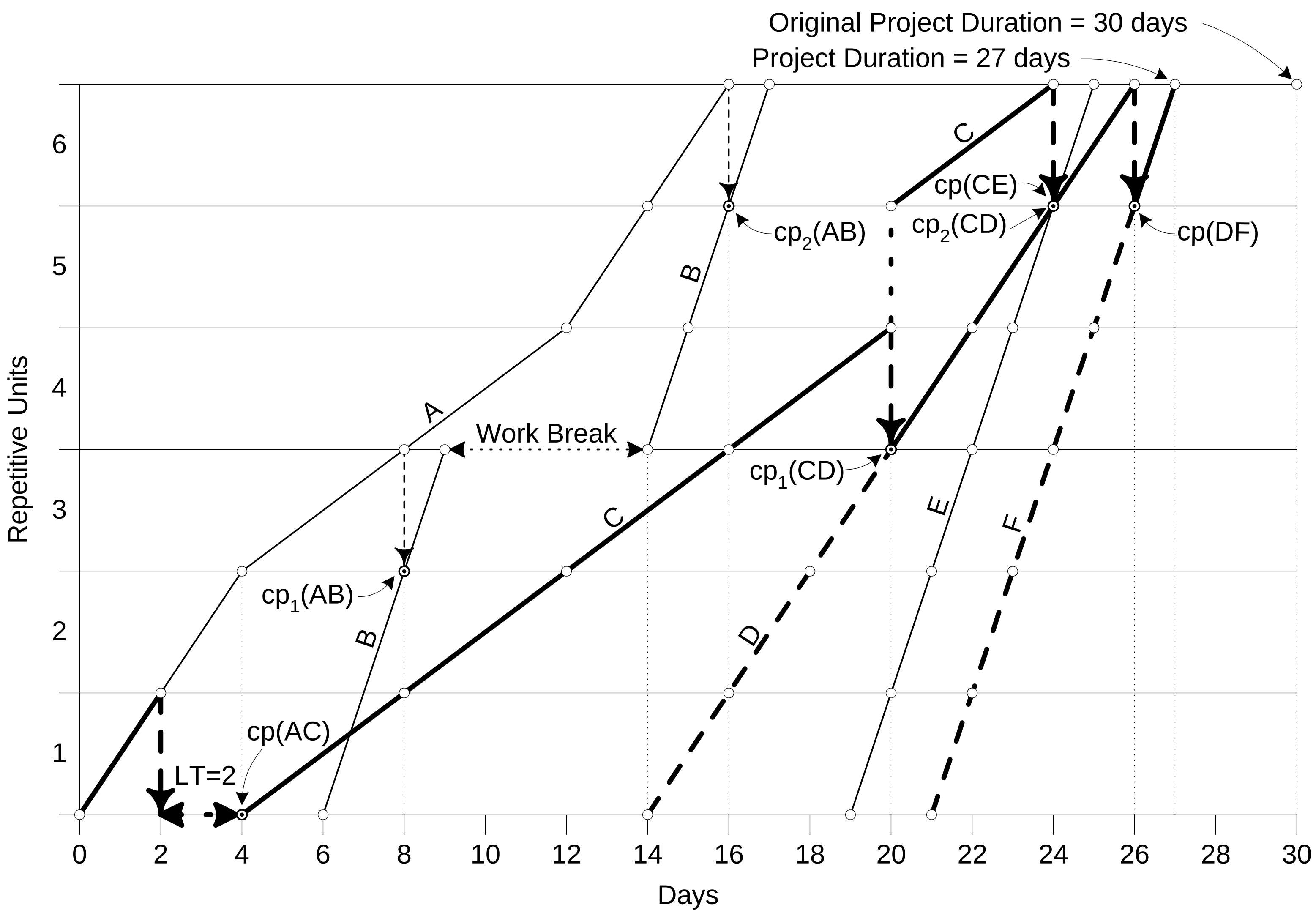


FIGURE 21
RSM DIAGRAM FOR A SIX UNIT PROJECT
WITH UNIT PRODUCTION RATE OF D LINE INCREASED

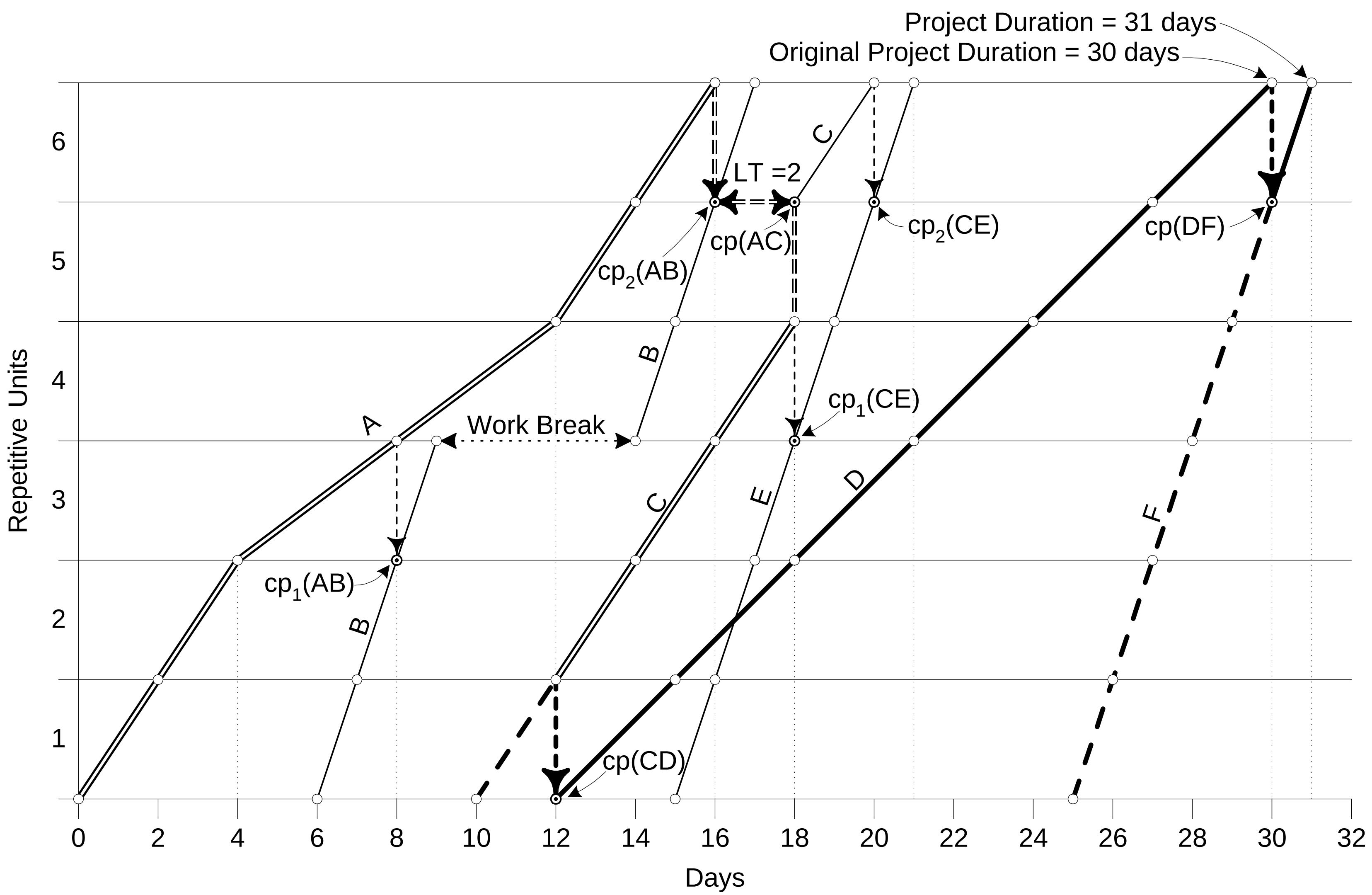


FIGURE 22
RSM DIAGRAM FOR A SIX UNIT PROJECT
WITH UNIT PRODUCTION RATE OF C LINE INCREASED

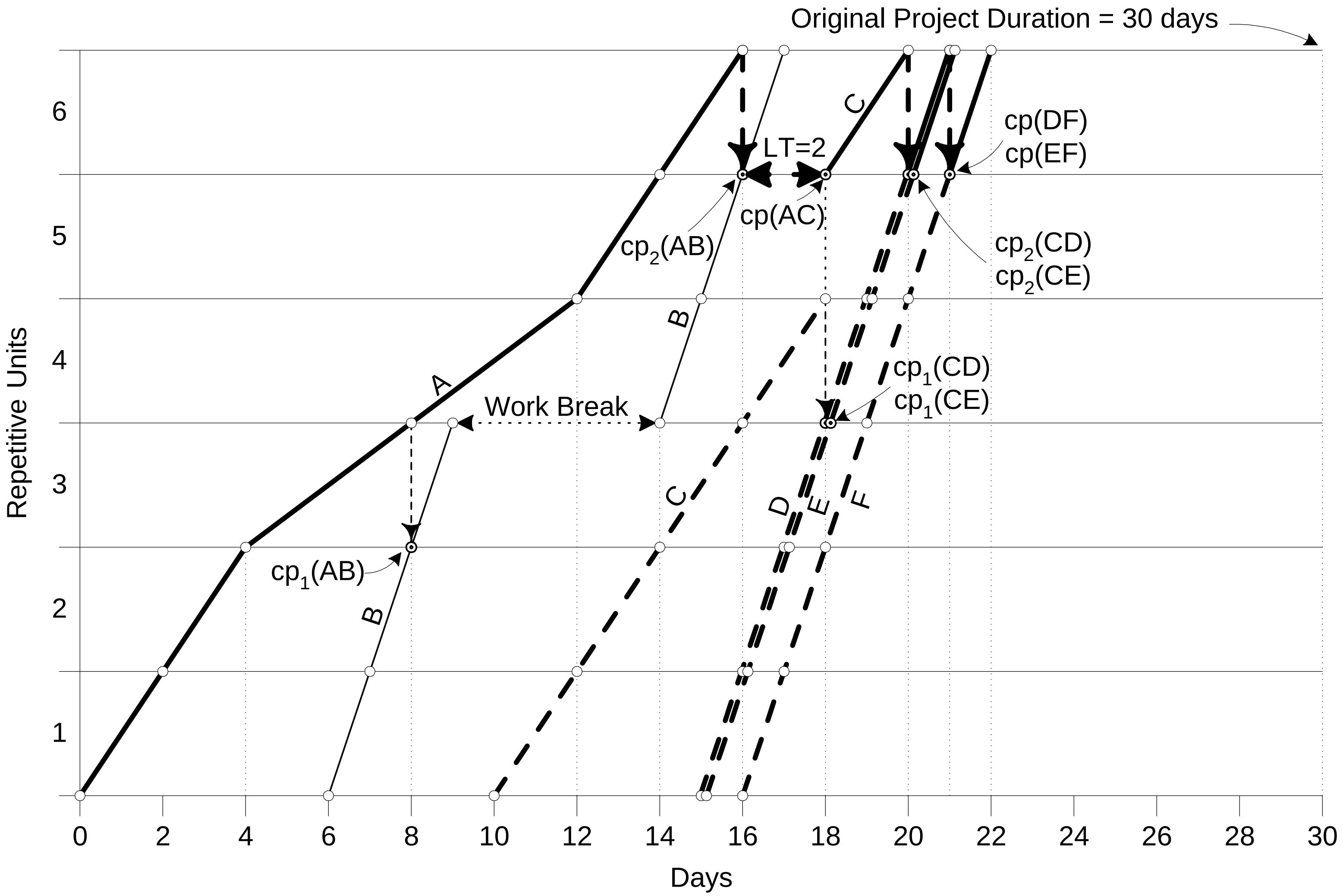


FIGURE 23
RSM DIAGRAM FOR A SIX UNIT PROJECT
WITH UNIT PRODUCTION RATE OF C AND D LINES INCREASED

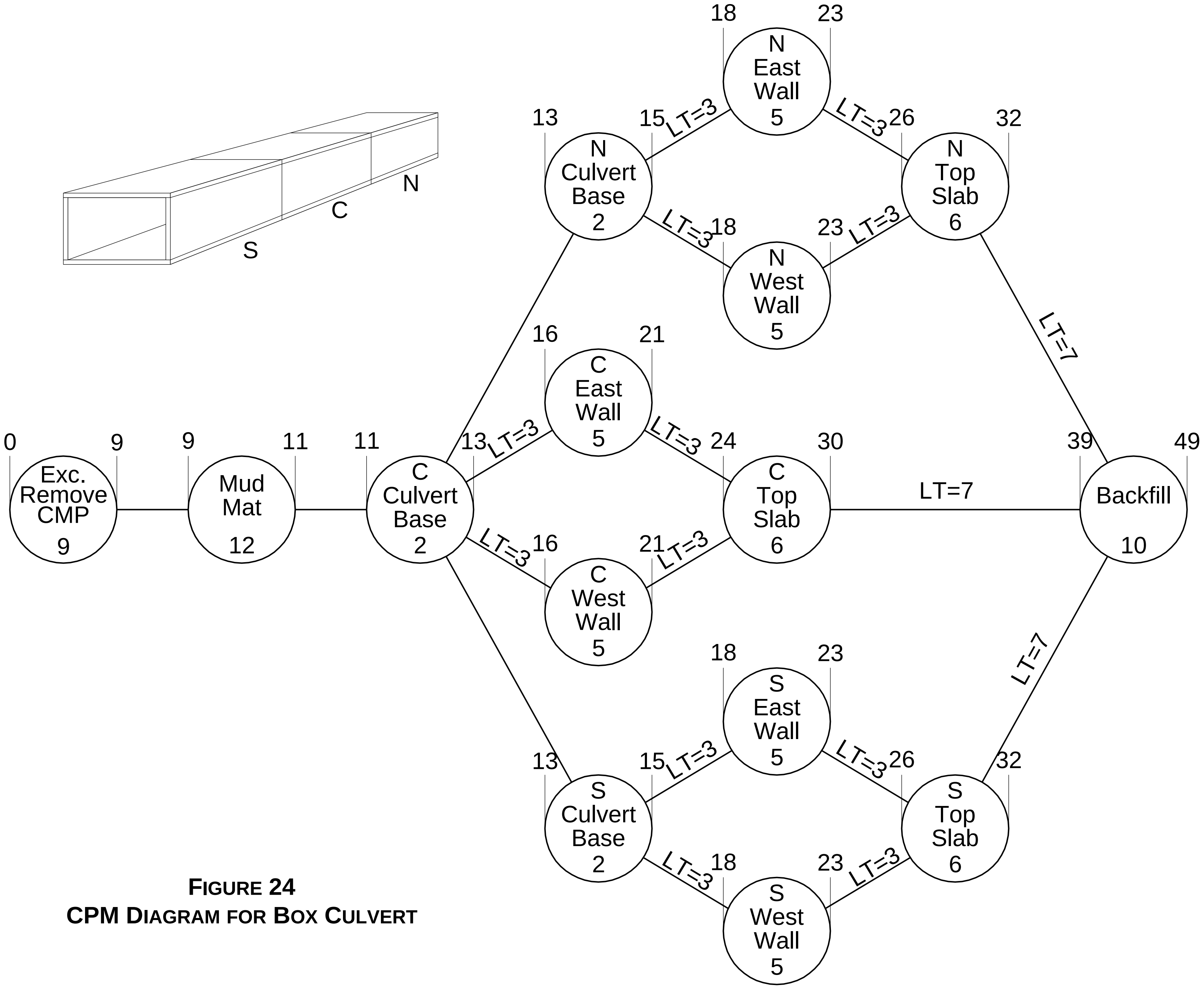


FIGURE 24
CPM DIAGRAM FOR BOX CULVERT

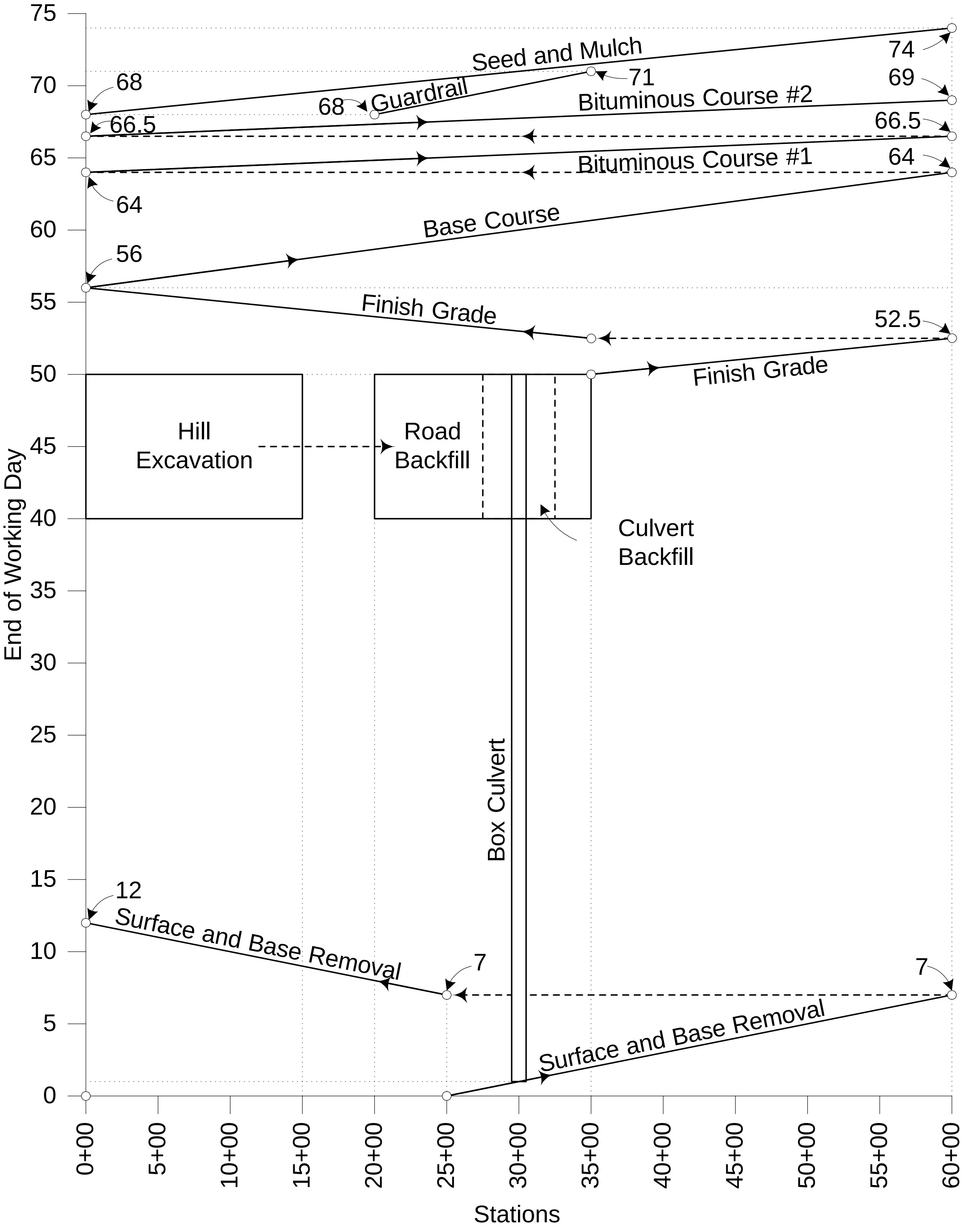


FIGURE 25
RSM SCHEDULE DIAGRAM FOR ROAD UPGRADE